DECAY OF A PION INTO A MUON AND A NEUTRINO

We can use the conservation of relativistic energy and momentum to analyze the interaction of elementary particles. For example, a pion at rest can decay into a muon and a neutrino. Conservation of energy and 3-momentum require

\[ E_\pi = m_\pi c^2 = E_\mu + E_\nu \]  \hspace{1cm} (1)

\[ p_\pi = 0 = p_\mu + p_\nu \]  \hspace{1cm} (2)

We can use the relation

\[ E^2 - p^2 c^2 = m^2 c^4 \]  \hspace{1cm} (3)

to relate energy and momentum. Assuming the neutrino is massless (it isn’t quite, but it’s close) we have

\[ E_\nu = c p_\nu \]  \hspace{1cm} (4)

while for the muon

\[ E_\mu = c \sqrt{p_\mu^2 + m_\mu^2 c^2} \]  \hspace{1cm} (5)

so

\[ m_\pi c = \sqrt{p_\mu^2 + m_\mu^2 c^2 + p_\nu} \]  \hspace{1cm} (6)

But \( p_\nu = -p_\mu \) from \( 2 \) so
where the last line follows from [5].

The velocity of the muon can be found from

\[ E_\mu = p^0 c \]
\[ = \frac{m_\mu c^2}{\sqrt{1-u^2/c^2}} \]
\[ \frac{m_\mu^2 + m_\pi^2 c^2}{2m_\pi c^2} = \frac{m_\mu c^2}{\sqrt{1-u^2/c^2}} \]
\[ u = c \sqrt{1 - \frac{4m_\pi^2 m_\mu^2}{(m_\pi^2 + m_\mu^2)^2}} \]
\[ = \frac{m_\mu^2}{m_\pi^2 + m_\mu^2 c} \]