DECAY OF A PION INTO A MUON AND A NEUTRINO

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 31.

[Griffiths's approach to the relativistic four-velocity is similar to that of Moore, although rather confusingly, he uses different notation (as well as keeping factors of c in the equations rather than setting $c=1$). To keep the notation consistent with Griffiths, I'll use his notation here, but anyone attempting to follow both books should beware.]

We can use the conservation of relativistic energy and momentum to analyze the interaction of elementary particles. For example, a pion at rest can decay into a muon and a neutrino. Conservation of energy and 3-momentum require

$$E_{\pi}=m_{\pi}c^{2} = E_{\mu}+E_{\nu} \tag{0.1}$$
$$\mathbf{p}_{\pi}=0 = \mathbf{p}_{\mu}+\mathbf{p}_{\nu} \tag{0.2}$$

We can use the relation

$$E^{2}-p^{2}c^{2}=m^{2}c^{4} \tag{0.3}$$

to relate energy and momentum. Assuming the neutrino is massless (it isn't quite, but it's close) we have

$$E_{\nu}=cp_{\nu} \tag{0.4}$$

while for the muon

$$E_{\mu}=c\sqrt{p_{\mu}^{2}+m_{\mu}^{2}c^{2}} \tag{0.5}$$

so

$$m_{\pi}c=\sqrt{p_{\mu}^{2}+m_{\mu}^{2}c^{2}}+p_{\nu} \tag{0.6}$$

But $p_\nu = -p_\mu$ from 0.2 so

$$m_\pi c = \sqrt{p_\mu^2 + m_\mu^2 c^2} - p_\mu \tag{0.7}$$

$$\sqrt{p_\mu^2 + m_\mu^2 c^2} = m_\pi c + p_\mu \tag{0.8}$$

$$p_\mu = \frac{m_\mu^2 - m_\pi^2}{2m_\pi} c \tag{0.9}$$

$$E_\mu = \frac{m_\mu^2 + m_\pi^2}{2m_\pi} c^2 \tag{0.10}$$

where the last line follows from 0.5.

The velocity of the muon can be found from

$$E_\mu = p^0 c \tag{0.11}$$

$$= \frac{m_\mu c^2}{\sqrt{1 - u^2/c^2}} \tag{0.12}$$

$$\frac{m_\mu^2 + m_\pi^2}{2m_\pi} c^2 = \frac{m_\mu c^2}{\sqrt{1 - u^2/c^2}} \tag{0.13}$$

$$u = c\sqrt{1 - \frac{4m_\pi^2 m_\mu^2}{\left(m_\pi^2 + m_\mu^2\right)}} \tag{0.14}$$

$$= \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2} c \tag{0.15}$$

PINGBACKS

Pingback: Collision of a pion and a proton
Pingback: Elastic collision of two identical particles