DECAY OF A PION INTO A MUON AND A NEUTRINO

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 31.

[Griffiths's approach to the relativistic four-velocity is similar to that of Moore, although rather confusingly, he uses different notation (as well as keeping factors of c in the equations rather than setting $c = 1$). To keep the notation consistent with Griffiths, I'll use his notation here, but anyone attempting to follow both books should beware.]

We can use the conservation of relativistic energy and momentum to analyze the interaction of elementary particles. For example, a pion at rest can decay into a muon and a neutrino. Conservation of energy and 3-momentum require

$$E_{\pi} = m_{\pi}c^2 = E_{\mu} + E_{\nu} \tag{1}$$

$$\mathbf{p}_{\pi} = 0 = \mathbf{p}_{\mu} + \mathbf{p}_{\nu} \tag{2}$$

We can use the relation

$$E^2 - p^2c^2 = m^2c^4 \tag{3}$$

to relate energy and momentum. Assuming the neutrino is massless (it isn't quite, but it's close) we have

$$E_{\nu} = cp_{\nu} \tag{4}$$

while for the muon

$$E_{\mu} = c\sqrt{p_{\mu}^2 + m_{\mu}^2c^2} \tag{5}$$

so

$$m_{\pi}c = \sqrt{p_{\mu}^2 + m_{\mu}^2c^2} + p_{\nu} \tag{6}$$

But $p_{\nu} = -p_{\mu}$ from 2 so

$$\begin{aligned}
m_\pi c &= \sqrt{p_\mu^2+m_\mu^2c^2}-p_\mu && (7)\\
\sqrt{p_\mu^2+m_\mu^2c^2} &= m_\pi c+p_\mu && (8)\\
p_\mu &= \frac{m_\mu^2-m_\pi^2}{2m_\pi}c && (9)\\
E_\mu &= \frac{m_\mu^2+m_\pi^2}{2m_\pi}c^2 && (10)
\end{aligned}$$

where the last line follows from 5.

The velocity of the muon can be found from

$$\begin{aligned}
E_\mu &= p^0c && (11)\\
&= \frac{m_\mu c^2}{\sqrt{1-u^2/c^2}} && (12)\\
\frac{m_\mu^2+m_\pi^2}{2m_\pi}c^2 &= \frac{m_\mu c^2}{\sqrt{1-u^2/c^2}} && (13)\\
u &= c\sqrt{1-\frac{4m_\pi^2m_\mu^2}{\left(m_\pi^2+m_\mu^2\right)}} && (14)\\
&= \frac{m_\pi^2-m_\mu^2}{m_\pi^2+m_\mu^2}c && (15)
\end{aligned}$$

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