DECAY OF A PION INTO A MUON AND A NEUTRINO

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 31.

[Griffiths's approach to the relativistic four-velocity is similar to that of Moore, although rather confusingly, he uses different notation (as well as keeping factors of c in the equations rather than setting $c = 1$). To keep the notation consistent with Griffiths, I'll use his notation here, but anyone attempting to follow both books should beware.]

We can use the conservation of relativistic energy and momentum to analyze the interaction of elementary particles. For example, a pion at rest can decay into a muon and a neutrino. Conservation of energy and 3-momentum require

$$E_\pi = m_\pi c^2 = E_\mu + E_\nu \tag{1}$$

$$\mathbf{p}_\pi = 0 = \mathbf{p}_\mu + \mathbf{p}_\nu \tag{2}$$

We can use the relation

$$E^2 - p^2c^2 = m^2c^4 \tag{3}$$

to relate energy and momentum. Assuming the neutrino is massless (it isn't quite, but it's close) we have

$$E_\nu = cp_\nu \tag{4}$$

while for the muon

$$E_\mu = c\sqrt{p_\mu^2 + m_\mu^2 c^2} \tag{5}$$

so

$$m_\pi c = \sqrt{p_\mu^2 + m_\mu^2 c^2} + p_\nu \tag{6}$$

But $p_\nu = -p_\mu$ from 2 so

$$m_\pi c = \sqrt{p_\mu^2 + m_\mu^2 c^2} - p_\mu \tag{7}$$

$$\sqrt{p_\mu^2 + m_\mu^2 c^2} = m_\pi c + p_\mu \tag{8}$$

$$p_\mu = \frac{m_\mu^2 - m_\pi^2}{2m_\pi} c \tag{9}$$

$$E_\mu = \frac{m_\mu^2 + m_\pi^2}{2m_\pi} c^2 \tag{10}$$

where the last line follows from 5.

The velocity of the muon can be found from

$$E_\mu = p^0 c \tag{11}$$

$$= \frac{m_\mu c^2}{\sqrt{1 - u^2/c^2}} \tag{12}$$

$$\frac{m_\mu^2 + m_\pi^2}{2m_\pi} c^2 = \frac{m_\mu c^2}{\sqrt{1 - u^2/c^2}} \tag{13}$$

$$u = c\sqrt{1 - \frac{4m_\pi^2 m_\mu^2}{\left(m_\pi^2 + m_\mu^2\right)}} \tag{14}$$

$$= \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 + m_\mu^2} c \tag{15}$$

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