# DECAY OF A PION INTO TWO PHOTONS

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 33.

Here's another example of using the conservation of relativistic energy and momentum, this time in the case of a pion with momentum $p=\frac{3}{4}mc$ decaying into two photons. One way of doing this problem is to start in the rest frame of the pion. In this frame, the two photons have equal energies and equal and opposite momenta, so

$$E_{\nu}=\frac{1}{2}mc^{2} \tag{0.1}$$

The momenta of the photons in the pion's rest frame are found from

$$\begin{aligned} E_{\nu} &= |p_{\nu}|c && (0.2)\\ p_{\nu} &= \pm\frac{1}{2}mc && (0.3) \end{aligned}$$

To transfer to the lab frame, we need to find the velocity of the pion:

$$\begin{aligned} \frac{mu}{\sqrt{1-u^{2}/c^{2}}}\equiv\gamma_{\pi}mu &= \frac{3}{4}mc && (0.4)\\ u &= \frac{3}{5}c && (0.5)\\ \gamma_{\pi} &= \frac{5}{4} && (0.6) \end{aligned}$$

Then the energies of the two photons can be found from a Lorentz transformation:

$$\begin{aligned} E_{lab} &= \gamma_{\pi}\left(E_{\nu}+\beta p_{\nu}\right) && (0.7)\\ &= \frac{5}{4}\left(\frac{1}{2}mc^{2}\pm\frac{3}{5}\left(\frac{1}{2}mc^{2}\right)\right) && (0.8)\\ &= \frac{1}{4}mc^{2},\ mc^{2} && (0.9) \end{aligned}$$

In the pion's frame, the two photons have equal energies and therefore equal wavelengths (from Planck's formula $E = h\nu$), while in the lab frame one photon's energy is half that of its energy in the pion's frame and the other is twice the energy, so one photon is red-shifted and the other is blue-shifted.