RELATIVE ENERGY IN A PARTICLE COLLIDER

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Post 34.

A technique used in particle accelerators such as the Large Hadron Collider to increase the relative energy of the incident particle on its target is to direct two opposing beams of particles at each other so that they collide head on.

In classical physics, the kinetic energy of a particle is $K=\frac{1}{2}mv^2$ so if a particle with this kinetic energy is fired at another identical particle at rest, this much energy is delivered into the collision. However, if we aim two opposing beams of particles, each with kinetic energy K at each other, the relative energy is increased by a factor of 4. This is because, classically, if we transform to the rest frame of one of the particles, the velocity of the other particle relative to the stationary one is $2v$, so K increases by a factor of $2^2=4$.

In relativity, however, we can get a much larger gain in relative energy. Suppose we fire two beams at each other so that, in the lab frame, each particle in both beams has a total energy E. If we now transform to the rest frame of one of the particles, the relative energy is

$$\bar{E}=\gamma(E+\beta cp) \tag{0.1}$$

where p is the particle's lab momentum, which we can get from the formula

$$c^2p^2=E^2-m^2c^4 \tag{0.2}$$

Also, since

$$\gamma=\frac{1}{\sqrt{1-\beta^2}}=\frac{E}{mc^2} \tag{0.3}$$

we have

$$\beta^2 = 1 - \frac{1}{\gamma^2} = 1 - \frac{m^2c^4}{E^2} = \frac{E^2 - m^2c^4}{E^2} \tag{0.4}$$

$$\beta cp = \sqrt{\frac{(E^2 - m^2c^4)^2}{E^2}} = \frac{E^2 - m^2c^4}{E} \tag{0.5}$$

Putting it all together, we get

$$\bar{E} = \frac{E}{mc^2}\left(E + \frac{E^2 - m^2c^4}{E}\right) \tag{0.6}$$

$$= \frac{2E^2}{mc^2} - mc^2 \tag{0.7}$$

That is, the relative energy increases as the *square* of the lab energy, which can lead to enormous gains. For example, if we fire 2 opposing beams of protons at each other at a lab energy of 30 GeV (the proton's rest mass is roughly 1 GeV), we find the relative energy is

$$\bar{E} = \frac{2 \times 30^2}{1} - 1 = 1799 \text{ GeV} \tag{0.8}$$

This is about 60 times the lab energy.