FORCE IN RELATIVITY

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Problem 12.36.

Force can be defined in relativity as the derivative of the spatial components of four-momentum with respect to ordinary (non-proper) time:

$$\mathbf{F}=\frac{d\mathbf{p}}{dt} \tag{1}$$

Superficially, this looks the same as Newton's second law, but in fact the formula for force is a bit more complex when written out in full. Using the definition of momentum, we have

$$\mathbf{F} = \frac{d}{dt}\left[\gamma m\mathbf{u}\right] \tag{2}$$

where

$$\gamma\equiv\frac{1}{\sqrt{1-u^{2}/c^{2}}} \tag{3}$$

We have

$$\begin{aligned}
\frac{d\gamma}{dt} &= \frac{1}{2c^{2}\left(1-u^{2}/c^{2}\right)^{3/2}}\frac{d\left(\mathbf{u}\cdot\mathbf{u}\right)}{dt} && (4)\\
&= \frac{\gamma^{3}}{2c^{2}}\left(2\mathbf{u}\cdot\mathbf{a}\right) && (5)\\
&= \frac{\mathbf{u}\cdot\mathbf{a}}{c^{2}\left(1-u^{2}/c^{2}\right)^{3/2}} && (6)
\end{aligned}$$

where $\mathbf{a}\equiv\dot{\mathbf{u}}$ is the acceleration.

Returning to 2, we have

$$\begin{aligned}
\mathbf{F} &= m\mathbf{u}\frac{d\gamma}{dt}+\gamma m\dot{\mathbf{u}} &(7)\\
&= \frac{m\left(\mathbf{u}\cdot\mathbf{a}\right)\mathbf{u}}{c^{2}\left(1-u^{2}/c^{2}\right)^{3/2}}+\frac{m\mathbf{a}}{\sqrt{1-u^{2}/c^{2}}} &(8)\\
&= \frac{m}{\sqrt{1-u^{2}/c^{2}}}\left[\mathbf{a}+\frac{\left(\mathbf{u}\cdot\mathbf{a}\right)\mathbf{u}}{c^{2}-u^{2}}\right] &(9)
\end{aligned}$$

This formula reduces to the familiar $\mathbf{F}=m\mathbf{a}$ in the limit of small $\mathbf{u}$. However, if we wish to retain a fixed acceleration as $u\to c$, the required force becomes infinite. Or looked at another way, if we want the force to remain finite as $u\to c$, the acceleration must drop to zero. In other words, it's impossible to accelerate an object with a non-zero rest mass to the speed of light.

PINGBACKS

Pingback: Outrunning a light ray
Pingback: Four-acceleration and Minkowski force
Pingback: Minkowski force
Pingback: Relativistic electromagnetic force
Pingback: Relativistic acceleration in terms of force