

FORCE IN RELATIVITY

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Problem 12.36.

Force can be defined in relativity as the derivative of the spatial components of four-momentum with respect to ordinary (non-proper) time:

$$\mathbf{F} = \frac{d\mathbf{p}}{dt} \quad (1)$$

Superficially, this looks the same as Newton's second law, but in fact the formula for force is a bit more complex when written out in full. Using the definition of momentum, we have

$$\mathbf{F} = \frac{d}{dt} [\gamma m \mathbf{u}] \quad (2)$$

where

$$\gamma \equiv \frac{1}{\sqrt{1 - u^2/c^2}} \quad (3)$$

We have

$$\frac{d\gamma}{dt} = \frac{1}{2c^2(1 - u^2/c^2)^{3/2}} \frac{d(\mathbf{u} \cdot \mathbf{u})}{dt} \quad (4)$$

$$= \frac{\gamma^3}{2c^2} (2\mathbf{u} \cdot \mathbf{a}) \quad (5)$$

$$= \frac{\mathbf{u} \cdot \mathbf{a}}{c^2(1 - u^2/c^2)^{3/2}} \quad (6)$$

where $\mathbf{a} \equiv \dot{\mathbf{u}}$ is the acceleration.

Returning to 2, we have

$$\mathbf{F} = m\mathbf{u}\frac{d\gamma}{dt} + \gamma m\dot{\mathbf{u}} \quad (7)$$

$$= \frac{m(\mathbf{u} \cdot \mathbf{a})\mathbf{u}}{c^2(1 - u^2/c^2)^{3/2}} + \frac{m\mathbf{a}}{\sqrt{1 - u^2/c^2}} \quad (8)$$

$$= \frac{m}{\sqrt{1 - u^2/c^2}} \left[\mathbf{a} + \frac{(\mathbf{u} \cdot \mathbf{a})\mathbf{u}}{c^2 - u^2} \right] \quad (9)$$

This formula reduces to the familiar $\mathbf{F} = m\mathbf{a}$ in the limit of small \mathbf{u} . However, if we wish to retain a fixed acceleration as $u \rightarrow c$, the required force becomes infinite. Or looked at another way, if we want the force to remain finite as $u \rightarrow c$, the acceleration must drop to zero. In other words, it's impossible to accelerate an object with a non-zero rest mass to the speed of light.

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