

## FORCE IN RELATIVITY

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Problem 12.36.

Force can be defined in relativity as the derivative of the spatial components of four-momentum with respect to ordinary (non-proper) time:

$$(1) \quad \mathbf{F} = \frac{d\mathbf{p}}{dt}$$

Superficially, this looks the same as Newton's second law, but in fact the formula for force is a bit more complex when written out in full. Using the definition of momentum, we have

$$(2) \quad \mathbf{F} = \frac{d}{dt}[\gamma m \mathbf{u}]$$

where

$$(3) \quad \gamma \equiv \frac{1}{\sqrt{1 - u^2/c^2}}$$

We have

$$(4) \quad \frac{d\gamma}{dt} = \frac{1}{2c^2(1 - u^2/c^2)^{3/2}} \frac{d(\mathbf{u} \cdot \mathbf{u})}{dt}$$

$$(5) \quad = \frac{\gamma^3}{2c^2} (2\mathbf{u} \cdot \mathbf{a})$$

$$(6) \quad = \frac{\mathbf{u} \cdot \mathbf{a}}{c^2(1 - u^2/c^2)^{3/2}}$$

where  $\mathbf{a} \equiv \dot{\mathbf{u}}$  is the acceleration.

Returning to 2, we have

$$\begin{aligned}
 (7) \quad \mathbf{F} &= m\mathbf{u}\frac{d\gamma}{dt} + \gamma m\dot{\mathbf{u}} \\
 (8) \quad &= \frac{m(\mathbf{u} \cdot \mathbf{a})\mathbf{u}}{c^2(1-u^2/c^2)^{3/2}} + \frac{m\mathbf{a}}{\sqrt{1-u^2/c^2}} \\
 (9) \quad &= \frac{m}{\sqrt{1-u^2/c^2}} \left[ \mathbf{a} + \frac{(\mathbf{u} \cdot \mathbf{a})\mathbf{u}}{c^2 - u^2} \right]
 \end{aligned}$$

This formula reduces to the familiar  $\mathbf{F} = m\mathbf{a}$  in the limit of small  $\mathbf{u}$ . However, if we wish to retain a fixed acceleration as  $u \rightarrow c$ , the required force becomes infinite. Or looked at another way, if we want the force to remain finite as  $u \rightarrow c$ , the acceleration must drop to zero. In other words, it's impossible to accelerate an object with a non-zero rest mass to the speed of light.

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