

## OUTRUNNING A LIGHT RAY

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Problem 12.37.

Even though nothing can move faster than light, it *is* possible for an object to arrive at any given location before a light ray, provided the object gets a bit of a head start. Suppose we have an object that is subject to a constant force in the  $+x$  direction. We've seen that (ordinary) force in relativity is the derivative of the spatial parts of the four-momentum with respect to ordinary time. In one dimension for a constant force we therefore have

$$(0.1) \quad \frac{dp}{dt} = F$$

$$(0.2) \quad p = Ft + C$$

where  $C$  is a constant of integration. If the object starts at  $t = 0$  at rest (in the lab frame), then  $C = 0$ , and

$$(0.3) \quad p = \frac{mu}{\sqrt{1 - u^2/c^2}} = Ft$$

which can be solved for the velocity  $u$  to give

$$(0.4) \quad u = \frac{F}{m} \frac{t}{\sqrt{1 + (Ft/mc)^2}}$$

This can be integrated again to get the position (assuming  $x = 0$  at  $t = 0$ ):

$$(0.5) \quad x(t) = \frac{F}{m} \int_0^t \frac{t' dt'}{\sqrt{1 + (Ft'/mc)^2}}$$

$$(0.6) \quad = \frac{mc^2}{F} \left[ \sqrt{1 + (Ft/mc)^2} - 1 \right]$$

We can rearrange this to get

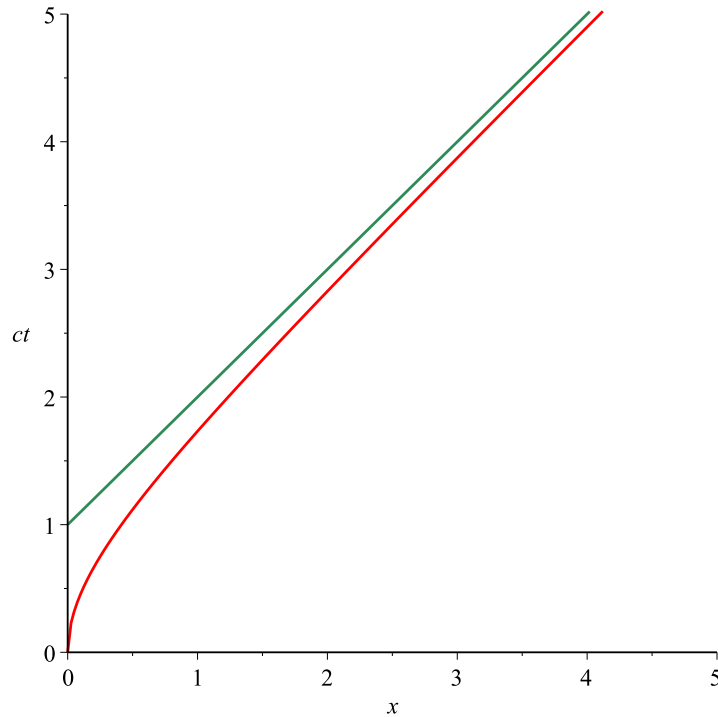
$$(0.7) \quad \left(\frac{F(ct)}{mc^2}\right)^2 - \left(\frac{Fx}{mc^2} + 1\right)^2 = 1$$

which is the equation of a hyperbola in the coordinates  $ct$  and  $x$ . The asymptotes are found by setting the RHS to zero, so we get

$$(0.8) \quad \frac{F(ct)}{mc^2} = \pm \left(\frac{Fx}{mc^2} + 1\right)$$

$$(0.9) \quad ct = \pm x \pm \frac{mc^2}{F}$$

For the case of motion in the  $+x$  direction, we take the plus sign, so the asymptote intersects the  $ct$  axis at  $ct = mc^2/F$ . We can plot this (for the case where  $F/mc^2 = 1$  in inverse distance units) on a spacetime diagram to get the red curve shown:



The green line is the asymptote, but it is also the world line of a light ray that leaves  $x = 0$  at  $ct = 1$ . Since it is the asymptote of the object's world line, the object will reach any given value of  $x$  *before* the light ray, so if an object is subjected to a constant force and given a head start (it starts moving at  $ct = 0$  and the light ray starts at  $ct = mc^2/F$ ) it will always be

ahead of the light ray (although admittedly not by much for large  $x$ ). This is true no matter how small the force, although the smaller the force, the larger the head start you'll need to stay ahead of the light ray.

#### PINGBACKS

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