OUTRUNNING A LIGHT RAY

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Problem 12.37.

Even though nothing can move faster than light, it *is* possible for an object to arrive at any given location before a light ray, provided the object gets a bit of a head start. Suppose we have an object that is subject to a constant force in the $+x$ direction. We've seen that (ordinary) force in relativity is the derivative of the spatial parts of the four-momentum with respect to ordinary time. In one dimension for a constant force we therefore have

$$\frac{dp}{dt} = F \tag{1}$$

$$p = Ft+C \tag{2}$$

where C is a constant of integration. If the object starts at $t=0$ at rest (in the lab frame), then $C=0$, and

$$p=\frac{mu}{\sqrt{1-u^2/c^2}}=Ft \tag{3}$$

which can be solved for the velocity u to give

$$u=\frac{F}{m}\frac{t}{\sqrt{1+(Ft/mc)^2}} \tag{4}$$

This can be integrated again to get the position (assuming $x=0$ at $t=0$):

$$x(t) = \frac{F}{m}\int_0^t\frac{t'\,dt'}{\sqrt{1+(Ft'/mc)^2}} \tag{5}$$

$$= \frac{mc^2}{F}\left[\sqrt{1+(Ft/mc)^2}-1\right] \tag{6}$$

We can rearrange this to get

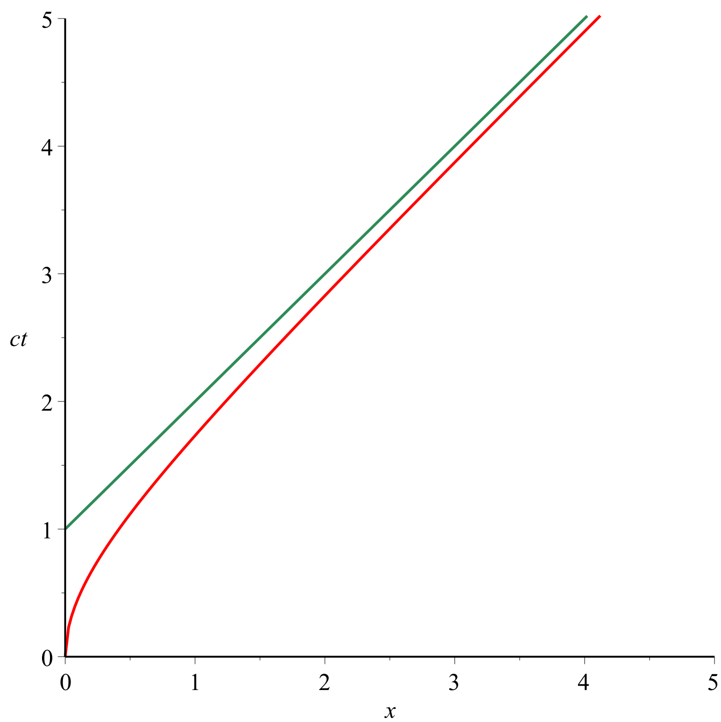
$$\left(\frac{F(ct)}{mc^2}\right)^2-\left(\frac{Fx}{mc^2}+1\right)^2=1 \tag{7}$$

which is the equation of a hyperbola in the coordinates ct and x. The asymptotes are found by setting the RHS to zero, so we get

$$\frac{F(ct)}{mc^2} = \pm\left(\frac{Fx}{mc^2}+1\right) \tag{8}$$

$$ct = \pm x \pm \frac{mc^2}{F} \tag{9}$$

For the case of motion in the $+x$ direction, we take the plus sign, so the asymptote intersects the ct axis at $ct = mc^2/F$. We can plot this (for the case where $F/mc^2 = 1$ in inverse distance units) on a spacetime diagram to get the red curve shown:



The green line is the asymptote, but it is also the world line of a light ray that leaves $x = 0$ at $ct = 1$. Since it is the asymptote of the object's world line, the object will reach any given value of x *before* the light ray, so if an object is subjected to a constant force and given a head start (it starts moving at $ct = 0$ and the light ray starts at $ct = mc^2/F$) it will always be ahead of the light ray (although admittedly not by much for large x). This is true no matter how small the force, although the smaller the force, the larger the head start you'll need to stay ahead of the light ray.

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