MINKOWSKI FORCE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Problem 12.39.

The Minkowski force is defined as the derivative of four-momentum with respect to proper time:

$$K^i = \frac{dp^i}{d\tau} \tag{0.1}$$

Its 0 component is therefore

$$K^0 = \frac{dp^0}{d\tau} = \frac{1}{c}\frac{dE}{d\tau} = \frac{\gamma}{c}\frac{dE}{dt} \tag{0.2}$$

We can relate K^0 to the ordinary force as follows. We have

$$\mathbf{F}\cdot\mathbf{u} = \frac{d\mathbf{p}}{dt}\cdot\mathbf{u} \tag{0.3}$$

$$= m\frac{d}{dt}\left(\frac{\mathbf{u}}{\sqrt{1-u^2/c^2}}\right)\cdot\mathbf{u} \tag{0.4}$$

Using

$$\frac{d}{dt}\left(\frac{1}{\sqrt{1-u^2/c^2}}\right) = \frac{\mathbf{u}\cdot\mathbf{a}}{c^2\left(1-u^2/c^2\right)^{3/2}} \tag{0.5}$$

we get

$$\mathbf{F}\cdot\mathbf{u} = \frac{m\mathbf{u}\cdot\mathbf{a}}{\left(1-u^2/c^2\right)^{3/2}} \tag{0.6}$$

$$= \frac{d}{dt}\left(\frac{mc^2}{\sqrt{1-u^2/c^2}}\right) \tag{0.7}$$

$$= \frac{dE}{dt} \tag{0.8}$$

So

$$K^0 = \frac{\gamma}{c}\mathbf{F}\cdot\mathbf{u} \tag{0.9}$$

For the spatial part of K^i we have

$$\mathbf{K}\cdot\mathbf{K} = \frac{d\mathbf{p}}{d\tau}\cdot\frac{d\mathbf{p}}{d\tau} \tag{0.10}$$

$$= \gamma^2\frac{d\mathbf{p}}{dt}\cdot\frac{d\mathbf{p}}{dt} \tag{0.11}$$

$$= \gamma^2 F^2 \tag{0.12}$$

Therefore

$$K_iK^i = -\left(K^0\right)^2 + \mathbf{K}\cdot\mathbf{K} \tag{0.13}$$

$$= \gamma^2\left(-\frac{1}{c^2}\left(\mathbf{F}\cdot\mathbf{u}\right)^2 + F^2\right) \tag{0.14}$$

$$= \gamma^2\left(1 - \frac{u^2\cos^2\theta}{c^2}\right)F^2 \tag{0.15}$$

where θ is the angle between $\mathbf{u}$ and $\mathbf{F}$.