

MINKOWSKI FORCE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Problem 12.39.

The Minkowski force is defined as the derivative of four-momentum with respect to proper time:

$$K^i = \frac{dp^i}{d\tau} \quad (1)$$

Its 0 component is therefore

$$K^0 = \frac{dp^0}{d\tau} = \frac{1}{c} \frac{dE}{d\tau} = \frac{\gamma}{c} \frac{dE}{dt} \quad (2)$$

We can relate K^0 to the ordinary force as follows. We have

$$\mathbf{F} \cdot \mathbf{u} = \frac{d\mathbf{p}}{dt} \cdot \mathbf{u} \quad (3)$$

$$= m \frac{d}{dt} \left(\frac{\mathbf{u}}{\sqrt{1-u^2/c^2}} \right) \cdot \mathbf{u} \quad (4)$$

Using

$$\frac{d}{dt} \left(\frac{1}{\sqrt{1-u^2/c^2}} \right) = \frac{\mathbf{u} \cdot \mathbf{a}}{c^2 (1-u^2/c^2)^{3/2}} \quad (5)$$

we get

$$\mathbf{F} \cdot \mathbf{u} = \frac{m\mathbf{u} \cdot \mathbf{a}}{(1-u^2/c^2)^{3/2}} \quad (6)$$

$$= \frac{d}{dt} \left(\frac{mc^2}{\sqrt{1-u^2/c^2}} \right) \quad (7)$$

$$= \frac{dE}{dt} \quad (8)$$

So

$$K^0 = \frac{\gamma}{c} \mathbf{F} \cdot \mathbf{u} \quad (9)$$

For the spatial part of K^i we have

$$\mathbf{K} \cdot \mathbf{K} = \frac{d\mathbf{p}}{d\tau} \cdot \frac{d\mathbf{p}}{d\tau} \quad (10)$$

$$= \gamma^2 \frac{d\mathbf{p}}{dt} \cdot \frac{d\mathbf{p}}{dt} \quad (11)$$

$$= \gamma^2 F^2 \quad (12)$$

Therefore

$$K_i K^i = -(K^0)^2 + \mathbf{K} \cdot \mathbf{K} \quad (13)$$

$$= \gamma^2 \left(-\frac{1}{c^2} (\mathbf{F} \cdot \mathbf{u})^2 + F^2 \right) \quad (14)$$

$$= \gamma^2 \left(1 - \frac{u^2 \cos^2 \theta}{c^2} \right) F^2 \quad (15)$$

where θ is the angle between \mathbf{u} and \mathbf{F} .