MINKOWSKI FORCE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Problem 12.39.

The Minkowski force is defined as the derivative of four-momentum with respect to proper time:

$$K^i = \frac{dp^i}{d\tau} \tag{1}$$

Its 0 component is therefore

$$K^0 = \frac{dp^0}{d\tau} = \frac{1}{c}\frac{dE}{d\tau} = \frac{\gamma}{c}\frac{dE}{dt} \tag{2}$$

We can relate K^0 to the ordinary force as follows. We have

$$\begin{aligned} \mathbf{F}\cdot\mathbf{u} &= \frac{d\mathbf{p}}{dt}\cdot\mathbf{u} &(3)\\ &= m\frac{d}{dt}\left(\frac{\mathbf{u}}{\sqrt{1-u^2/c^2}}\right)\cdot\mathbf{u} &(4)\end{aligned}$$

Using

$$\frac{d}{dt}\left(\frac{1}{\sqrt{1-u^2/c^2}}\right) = \frac{\mathbf{u}\cdot\mathbf{a}}{c^2\left(1-u^2/c^2\right)^{3/2}} \tag{5}$$

we get

$$\begin{aligned} \mathbf{F}\cdot\mathbf{u} &= \frac{m\mathbf{u}\cdot\mathbf{a}}{\left(1-u^2/c^2\right)^{3/2}} &(6)\\ &= \frac{d}{dt}\left(\frac{mc^2}{\sqrt{1-u^2/c^2}}\right) &(7)\\ &= \frac{dE}{dt} &(8)\end{aligned}$$

So

$$K^0 = \frac{\gamma}{c}\mathbf{F}\cdot\mathbf{u} \tag{9}$$

For the spatial part of K^i we have

$$\begin{aligned}
\mathbf{K}\cdot\mathbf{K} &= \frac{d\mathbf{p}}{d\tau}\cdot\frac{d\mathbf{p}}{d\tau} && (10)\\
&= \gamma^2\frac{d\mathbf{p}}{dt}\cdot\frac{d\mathbf{p}}{dt} && (11)\\
&= \gamma^2 F^2 && (12)
\end{aligned}$$

Therefore

$$\begin{aligned}
K_i K^i &= -\left(K^0\right)^2 + \mathbf{K}\cdot\mathbf{K} && (13)\\
&= \gamma^2\left(-\frac{1}{c^2}\left(\mathbf{F}\cdot\mathbf{u}\right)^2 + F^2\right) && (14)\\
&= \gamma^2\left(1 - \frac{u^2\cos^2\theta}{c^2}\right)F^2 && (15)
\end{aligned}$$

where θ is the angle between $\mathbf{u}$ and $\mathbf{F}$.