

## MINKOWSKI FORCE

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Problem 12.39.

The Minkowski force is defined as the derivative of four-momentum with respect to proper time:

$$(1) \quad K^i = \frac{dp^i}{d\tau}$$

Its 0 component is therefore

$$(2) \quad K^0 = \frac{dp^0}{d\tau} = \frac{1}{c} \frac{dE}{d\tau} = \frac{\gamma dE}{c dt}$$

We can relate  $K^0$  to the ordinary force as follows. We have

$$(3) \quad \mathbf{F} \cdot \mathbf{u} = \frac{d\mathbf{p}}{dt} \cdot \mathbf{u}$$

$$(4) \quad = m \frac{d}{dt} \left( \frac{\mathbf{u}}{\sqrt{1 - u^2/c^2}} \right) \cdot \mathbf{u}$$

Using

$$(5) \quad \frac{d}{dt} \left( \frac{1}{\sqrt{1 - u^2/c^2}} \right) = \frac{\mathbf{u} \cdot \mathbf{a}}{c^2 (1 - u^2/c^2)^{3/2}}$$

we get

$$(6) \quad \mathbf{F} \cdot \mathbf{u} = \frac{m\mathbf{u} \cdot \mathbf{a}}{(1 - u^2/c^2)^{3/2}}$$

$$(7) \quad = \frac{d}{dt} \left( \frac{mc^2}{\sqrt{1 - u^2/c^2}} \right)$$

$$(8) \quad = \frac{dE}{dt}$$

So

$$(9) \quad K^0 = \frac{\gamma}{c} \mathbf{F} \cdot \mathbf{u}$$

For the spatial part of  $K^i$  we have

$$(10) \quad \mathbf{K} \cdot \mathbf{K} = \frac{d\mathbf{p}}{d\tau} \cdot \frac{d\mathbf{p}}{d\tau}$$

$$(11) \quad = \gamma^2 \frac{d\mathbf{p}}{dt} \cdot \frac{d\mathbf{p}}{dt}$$

$$(12) \quad = \gamma^2 F^2$$

Therefore

$$(13) \quad K_i K^i = -(K^0)^2 + \mathbf{K} \cdot \mathbf{K}$$

$$(14) \quad = \gamma^2 \left( -\frac{1}{c^2} (\mathbf{F} \cdot \mathbf{u})^2 + F^2 \right)$$

$$(15) \quad = \gamma^2 \left( 1 - \frac{u^2 \cos^2 \theta}{c^2} \right) F^2$$

where  $\theta$  is the angle between  $\mathbf{u}$  and  $\mathbf{F}$ .