RELATIVISTIC INVARIANTS INVOLVING ELECTROMAGNETIC FIELDS

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References: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Problem 12.46.

Using the transformation equations for electromagnetic fields, we can show that there are a couple of quantities that are invariant under transformations between inertial frames.

The transformation equations are

$$\bar{E}_x = E_x \tag{1}$$

$$\bar{E}_y = \gamma(E_y + vB_z) \tag{2}$$

$$\bar{E}_z = \gamma(E_z - vB_y) \tag{3}$$

$$\bar{B}_x = B_x \tag{4}$$

$$\bar{B}_y = \gamma\left(B_y - \frac{v}{c^2}E_z\right) \tag{5}$$

$$\bar{B}_z = \gamma\left(B_z + \frac{v}{c^2}E_y\right) \tag{6}$$

First, we calculate the dot product $\mathbf{E}\cdot\mathbf{B}$ and transform it:

$$\bar{\mathbf{E}}\cdot\bar{\mathbf{B}} = E_xB_x + \gamma^2\left[(E_y + vB_z)\left(B_y - \frac{v}{c^2}E_z\right) + (E_z - vB_y)\left(B_z + \frac{v}{c^2}E_y\right)\right] \tag{7}$$

$$= E_xB_x + \gamma^2\left[(E_yB_y + E_zB_z)\left(1 - \frac{v^2}{c^2}\right)\right] \tag{8}$$

$$= E_xB_x + E_yB_y + E_zB_z \tag{9}$$

$$= \mathbf{E}\cdot\mathbf{B} \tag{10}$$

Second, we can find another invariant:

$$\bar{E}^2 - c^2\bar{B}^2 = E_x^2 - c^2B_x^2 + \gamma^2\left[(E_y + vB_z)^2 + (E_z - vB_y)^2 - c^2\left[\left(B_y - \frac{v}{c^2}E_z\right)^2 + \left(B_z + \frac{v}{c^2}E_y\right)^2\right]\right] \tag{11}$$

$$= E_x^2 - c^2B_x^2 + \gamma^2\left[\left(E_y^2 + E_z^2 - c^2B_y^2 - c^2B_z^2\right)\left(1 - \frac{v^2}{c^2}\right)\right] \tag{12}$$

$$= E^2 - c^2B^2 \tag{13}$$

These invariants put constraints on the forms given electric and magnetic fields can have in different frames. For example, if $\mathbf{B} = 0$ and $\mathbf{E} \neq 0$ in one frame, then $E^2 - c^2B^2 > 0$ in all frames, so it's impossible to find a frame in which $\mathbf{E} = 0$. The first invariant also tells us that if $\mathbf{E}$ and $\mathbf{B}$ are perpendicular in one frame (as they are in an electromagnetic wave, for example) then they are perpendicular in all frames.

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