ELECTROMAGNETIC MINKOWSKI FORCE

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Reference: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Problem 12.54.

The Minkowski force is defined in general by

$$\mathbf{K}=\frac{d\mathbf{p}}{d\tau}=\frac{1}{\sqrt{1-\beta^{2}}}\frac{d\mathbf{p}}{dt}=\frac{1}{\sqrt{1-\beta^{2}}}\mathbf{F} \tag{1}$$

where $\mathbf{p}$ is the spatial part of the four-momentum. Griffiths shows in his section 12.3.4 that the Minkowski force due to electromagnetic fields is

$$K^{i}=q\eta_{j}F^{ij} \tag{2}$$

where the electromagnetic field tensor is (with $c=1$):

$$F^{ij}=\left[\begin{array}{cccc} 0 & E_{x} & E_{y} & E_{z}\\ -E_{x} & 0 & B_{z} & -B_{y}\\ -E_{y} & -B_{z} & 0 & B_{x}\\ -E_{z} & B_{y} & -B_{x} & 0 \end{array}\right] \tag{3}$$

and the proper velocity is

$$\eta_{i}=\frac{dx_{i}}{d\tau}=\frac{u_{i}}{\sqrt{1-\beta^{2}}} \tag{4}$$

Griffiths shows that the spatial components of K^{i} work out to

$$\mathbf{K}=\frac{q}{\sqrt{1-\beta^{2}}}\left(\mathbf{E}+\mathbf{u}\times\mathbf{B}\right)=\frac{1}{\sqrt{1-\beta^{2}}}\mathbf{F} \tag{5}$$

so the relation 1 between $\mathbf{K}$ and $\mathbf{F}$ is correct.

To see what the time component of 2 gives us, we can just work it out by reading off the first row of F^{ij}:

$$
\begin{aligned}
K^0 &= q\eta_j F^{0j} && (6)\\
&= \frac{q}{\sqrt{1-\beta^2}}\left(u_x E_x + u_y E_y + u_z E_z\right) && (7)\\
&= \frac{q}{\sqrt{1-\beta^2}}\mathbf{u}\cdot\mathbf{E} && (8)\\
&= \frac{1}{\sqrt{1-\beta^2}}\mathbf{u}\cdot\mathbf{F}_E && (9)
\end{aligned}
$$

The term $\mathbf{u}\cdot\mathbf{F}_E$ is the rate at which the electric field does work on the charge as it moves along (remember that because the direction of motion is always perpendicular to the force exerted by a magnetic field, **B** never does any work, so this is the total work done by the electromagnetic field).