COLLISION OF A PION AND A PROTON

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Reference: Griffiths, David J. (2007), Introduction to Electrodynamics, 3rd Edition; Pearson Education - Chapter 12, Problem 12.58.

As another example of using conservation of energy and momentum to work out the kinematics of particle collisions, suppose we fire a pion at a proton at rest. One possible outcome of such a collision is the conversion of the pion and proton into kappa and sigma particles, but this can only occur if the momentum of the pion is high enough, since the rest energies of the kappa plus sigma are greater than those of the pion plus proton. We can find the minimum pion momentum (called the threshold momentum), as measured in the lab, at which this reaction can occur. It's easiest to convert to the centre of momentum frame to do the calculations and then convert back at the end.

In the centre of momentum frame, the pion and proton head towards each other with equal and opposite momenta, so using the usual relativistic notation, and expressing rest energy in MeV (so we can ignore the c^2 factor):

$$\gamma_{\pi}\beta_{\pi}m_{\pi}=\gamma_{p}\beta_{p}m_{p} \tag{1}$$

At the threshold momentum, the pion and proton collide and produce a K and Σ at rest, so from conservation of energy

$$\gamma_{\pi}m_{\pi}+\gamma_{p}m_{p}=m_{K}+m_{\Sigma} \tag{2}$$

Using Griffiths's approximate values for the rest energies, we have (in MeV)

$$m_{\pi} = 150 \tag{3}$$
$$m_{p} = 900 \tag{4}$$
$$m_{K} = 500 \tag{5}$$
$$m_{\Sigma} = 1200 \tag{6}$$

so from 1 and 2

$$\begin{aligned}
150\gamma_\pi\beta_\pi &= 900\gamma_p\beta_p && (7)\\
\gamma_\pi\beta_\pi &= 6\gamma_p\beta_p && (8)\\
150\gamma_\pi + 900\gamma_p &= 1700 && (9)\\
\gamma_\pi + 6\gamma_p &= \frac{34}{3} && (10)
\end{aligned}$$

From these equations, we get

$$\begin{aligned}
\gamma_\pi^2 &= \left(\frac{34}{3} - 6\gamma_p\right)^2 = \frac{1}{1-\beta_\pi^2} && (11)\\
\beta_\pi^2 &= 1 - \left(\frac{34}{3} - 6\gamma_p\right)^{-2} && (12)\\
(\gamma_\pi\beta_\pi)^2 &= \left(\frac{34}{3} - 6\gamma_p\right)^2 - 1 && (13)\\
&= 36\gamma_p^2\beta_p^2 && (14)
\end{aligned}$$

where we used 8 to get the last line. We can now solve the last two equations to find γ_p:

$$\begin{aligned}
36\gamma_p^2\beta_p^2 &= \left(\frac{34}{3} - 6\gamma_p\right)^2 - 1 && (15)\\
&= \left(\frac{34}{3}\right)^2 - 1 - 136\gamma_p + 36\gamma_p^2 && (16)\\
0 &= \left(\frac{34}{3}\right)^2 - 1 - 136\gamma_p + 36\gamma_p^2\left(1-\beta_p^2\right) && (17)\\
0 &= \left(\frac{34}{3}\right)^2 - 1 + 36 - 136\gamma_p && (18)\\
\gamma_p &= 1.202 && (19)\\
\beta_p &= 0.555 && (20)
\end{aligned}$$

We can now get the values for the pion in the centre of momentum frame from 11 and 12:

$$\gamma_\pi = \frac{34}{3} - 6\gamma_p = 4.123 \tag{21}$$

$$\beta_\pi = \sqrt{1 - \left(\frac{34}{3} - 6\gamma_p\right)^{-2}} = 0.970 \tag{22}$$

The speed of the proton in the centre of momentum frame is also the speed of the centre of momentum frame relative to the lab frame, so we can use a Lorentz transformation on the pion's four-momentum to get back to the lab frame:

$$p_\pi^1 = \gamma_p \left(\bar{p}_\pi^1 + \beta_p \bar{p}_\pi^0\right) \tag{23}$$

$$= 1.202\,(150\gamma_\pi\beta_\pi + 0.555 \times 150\gamma_\pi) \tag{24}$$

$$= 1133 \text{ MeV}/c \tag{25}$$

Notice that we must use γ_p and β_p (that is, the values for the *proton*, not the pion) in doing the Lorentz transformation, since it's the speed of the proton that determines the relative speed of the two frames.