

## TRIANGULAR WAVE FUNCTION: PROBABILITIES

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Post date: 23 Jun 2015.

Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 1.4.

The square modulus of the wave function which is the solution to the Schrödinger equation is interpreted as a probability density. As an example consider the wave function given by

$$\Psi(x,0) = \begin{cases} A\frac{x}{a} & 0 \leq x \leq a \\ A\frac{b-x}{b-a} & a \leq x \leq b \\ 0 & \text{otherwise} \end{cases} \quad (1)$$

We can normalize  $\Psi$  by requiring

$$\int_0^b |\Psi|^2 dx = 1 \quad (2)$$

Plugging in the formula and doing the integral gives

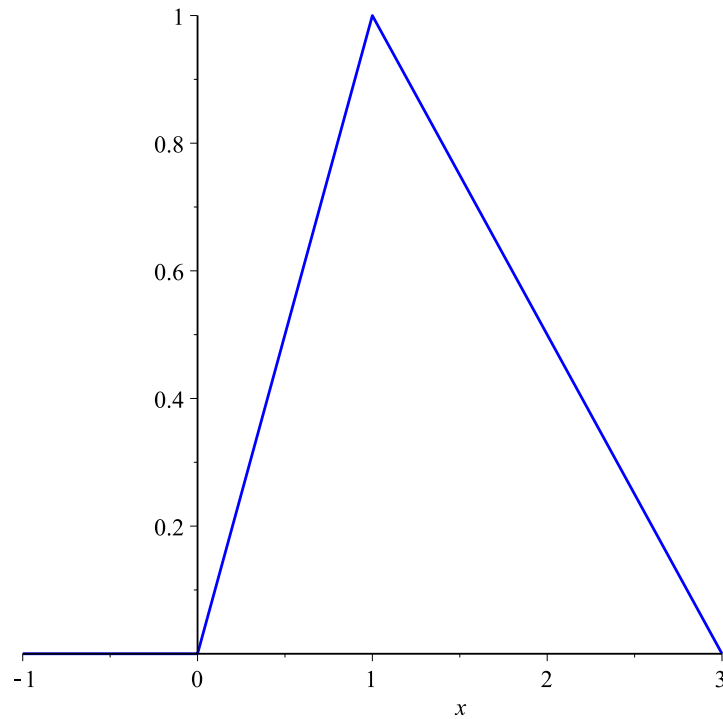
$$\int_0^b |\Psi|^2 dx = |A|^2 \left[ \int_0^a \frac{x^2}{a^2} dx + \int_a^b \left( \frac{b-x}{b-a} \right)^2 dx \right] \quad (3)$$

$$= |A|^2 \frac{b}{3} \quad (4)$$

$$A = \sqrt{\frac{3}{b}} \quad (5)$$

where we've taken the positive real root for  $A$ . Note that  $A$  could also be multiplied by a phase factor  $e^{i\delta}$  for any real  $\delta$  without affecting normalization. This can be important in some applications where we need to add together wave functions.

Given this value for  $A$ , we can plot 1. Here, we've taken  $a = 1$  and  $b = 3$ :



Since  $\Psi$  has its maximum at  $x = a$ , that is where the particle is most likely to be found. The probability of the particle being found to the left of  $x = a$  is

$$P_{x < a} = \frac{3}{b} \int_0^a \frac{x^2}{a^2} dx = \frac{a}{b} \quad (6)$$

If  $b = a$ , then  $\Psi$  drops to zero at  $x = a$  so  $P_{x < a} = 1$ . If  $b = 2a$ , then  $\Psi$  is an isosceles triangle symmetric about  $x = a$  so  $P_{x < a} = \frac{1}{2}$ .

The expectation value of  $x$  is

$$\langle x \rangle = \int x |\Psi|^2 dx = \frac{a}{2} + \frac{b}{4} \quad (7)$$

where we used Maple to simplify the integration. If  $b = 2a$ , then  $\langle x \rangle = a$  as expected.