

DELTA FUNCTION WELL: STATISTICS

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 1.5.

The delta function well gives rise to a wave function that decays exponentially either side of the delta function:

$$(1) \quad \Psi(x,t) = Ae^{-\lambda|x|}e^{-i\omega t}$$

We can normalize Ψ in the usual way:

$$(2) \quad \int_{-\infty}^{\infty} |\Psi|^2 dx = |A|^2 \left[\int_{-\infty}^0 e^{2\lambda x} dx + \int_0^{\infty} e^{-2\lambda x} dx \right]$$

$$(3) \quad = 2|A|^2 \int_0^{\infty} e^{-2\lambda x} dx$$

$$(4) \quad = \frac{|A|^2}{\lambda}$$

$$(5) \quad A = \sqrt{\lambda}$$

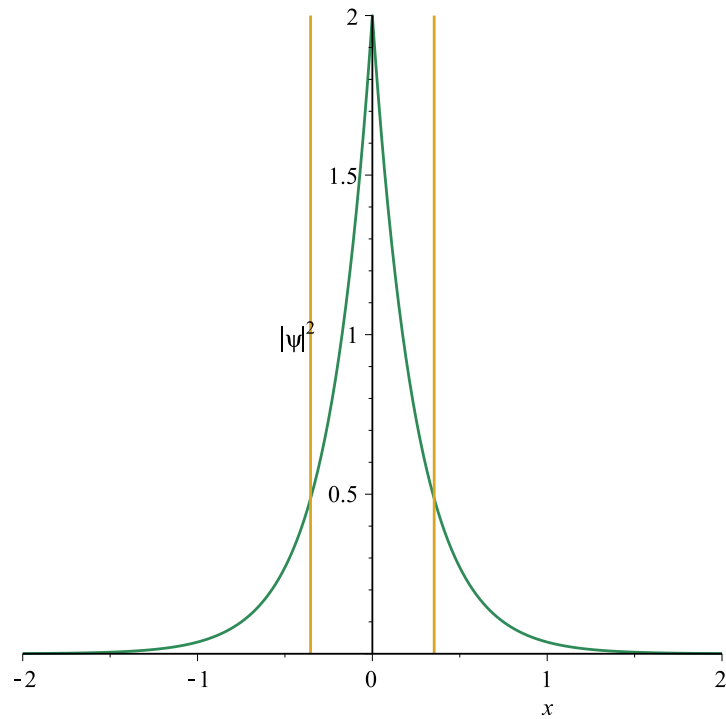
By symmetry, $\langle x \rangle = 0$ and

$$(6) \quad \langle x^2 \rangle = 2\lambda \int_0^{\infty} x^2 e^{-2\lambda x} dx = \frac{1}{2\lambda^2}$$

Therefore

$$(7) \quad \sigma = \sqrt{\langle x^2 \rangle - \langle x \rangle^2} = \frac{1}{\sqrt{2}\lambda}$$

A plot of $|\Psi|^2$ is shown, with vertical yellow lines indicating $x = \pm \frac{1}{\sqrt{2}\lambda}$, for the case $\lambda = 2$:



The probability that the particle lies outside $x = \pm \frac{1}{\sqrt{2}\lambda}$ is

$$(8) \quad P_{|x|>\sigma} = 2\lambda \int_{1/\sqrt{2}\lambda}^{\infty} e^{-2\lambda x} dx = \frac{1}{e^{\sqrt{2}}} = 0.2431$$

In this case, the probability of x being greater than one standard deviation is a constant, independent of λ .