SCHRÖDINGER EQUATION - A FEW THEOREMS

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.1.

We've seen that the time-independent Schrödinger equationcan, in the case where the potential $V(x)$ is independent of time, be separated into two ordinary differential equations, one in the space coordinate x and the other in the time t. The two equations are

$$\begin{aligned} -\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) &= E\psi(x) \quad (1) \\ i\hbar\frac{d\Xi(t)}{dt} &= E\Xi(t) \quad (2) \end{aligned}$$

where E is the separation constant.

The second one can be solved to get

$$\Xi(t) = Ce^{-iEt/\hbar} \quad (3)$$

If the wave function is normalizable, then the separation constant E must be real. Proof: Suppose $E = E_0 + i\Gamma$. Then $\Psi(x,t) = \psi(x)e^{-iEt/\hbar} = \psi(x)e^{-iE_0t/\hbar}e^{\Gamma t/\hbar}$. To normalize, we must have $\int |\Psi|^2 dx = 1$, so $e^{2\Gamma t/\hbar}\int |\psi|^2 dx = 1$. Since this must be true for all times, we must have $\Gamma = 0$.

The time-independent wave function can always be taken to be real. This follows from the fact that the Schrodinger equation is linear, so if the wave function is complex, its real and imaginary parts will satisfy the equation separately.

If the potential $V(x) = V(-x)$ (is even), then $\psi(x)$ can be taken as even or odd. Follows by considering the Schrödinger equation with x replaced by $-x$:

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(-x)}{dx^2} + V(x)\psi(-x) = E\psi(-x) \quad (4)$$

Thus $\psi(-x)$ satisfies the same equation as $\psi(x)$ for an even potential. Therefore, the two linear combinations $\psi_{even} = \psi(x) + \psi(-x)$ and $\psi_{odd} =$

$\psi(x) - \psi(-x)$ also satisfy the equation. The general solution can then be built from a linear combination of even and odd functions.

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