INFINITE SQUARE WELL - MINIMUM ENERGY

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.3.

We've seen that the energy of a system must always be greater than the minimum of the potential function. As a specific example of this we can look at the Schrödinger equation for the square well, between $x=0$ and $x=a$:

$$\frac{d^2\psi}{dx^2}=-\frac{2m}{\hbar^2}E\psi \tag{1}$$

If $E=0$, $\psi''=0$. Integrating gives $\psi=Ax+B$. Attempting to satisfy the boundary conditions, we get $\psi(0)=0$ giving $B=0$. Then the condition $\psi(a)=0$ gives $A=0$, thus $\psi(x)=0$ and cannot be normalized.

If $E<0$, we solve the equation

$$\psi''=-\frac{2mE}{\hbar^2}\psi\equiv k^2\psi \tag{2}$$

with $k=\sqrt{-2mE/\hbar^2}$. Since $E<0$, k is real. The general solution is $\psi(x)=Ae^{kx}+Be^{-kx}$. Applying boundary conditions, we get $\psi(0)=0=A+B$, so $A=-B$. At $x=a$, we have

$$\begin{aligned}
\psi(a) &= 0 && (3)\\
&= Ae^{ka}+Be^{-ka} && (4)\\
&= AC+\frac{B}{C} && (5)\\
&= A\left(C-\frac{1}{C}\right) && (6)
\end{aligned}$$

where $C\equiv e^{ka}>0$. Since C is strictly positive (being an exponential) the only way we can get $C-1/C=0$ is for $C=1$, implying $ka=0$. However, neither k nor a is zero here, so $C\neq 1$, so $A=0=B$ and $\psi(x)=0$ again.

Thus if $E \leq 0$, the wave function cannot be normalized *and* satisfy the boundary conditions.