INFINITE SQUARE WELL - UNCERTAINTY PRINCIPLE

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.4.

We can calculate the mean values of position and momentum and verify the uncertainty principle for the infinite square well. The Schrödinger equation for the square wellis, between $x=0$ and $x=a$:

$$\frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}E\psi \tag{0.1}$$

The stationary states of the infinite square well are given by

$$\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)e^{-i(n^2\pi^2\hbar/2ma^2)t} \tag{0.2}$$

for $0\le x\le a$.

For x we have

$$\langle x\rangle = \frac{2}{a}\int_0^a x\sin^2(n\pi x/a)dx = a/2 \tag{0.3}$$

$$\left\langle x^2\right\rangle = \frac{2}{a}\int_0^a x^2\sin^2(n\pi x/a)dx = a^2\left(\frac{1}{3}-\frac{1}{2n^2\pi^2}\right) \tag{0.4}$$

$$\sigma_x^2 = \left\langle x^2\right\rangle - \langle x\rangle^2 = a^2\left(\frac{1}{12}-\frac{1}{2n^2\pi^2}\right) = \frac{n^2\pi^2-6}{12n^2\pi^2}a^2 \tag{0.5}$$

For the momentum$p$ we have

$$\langle p\rangle = \frac{2\hbar}{ai}\int_0^a \sin(n\pi x/a)(n\pi/a)\cos(n\pi x/a)dx = 0 \tag{0.6}$$

$$\left\langle p^2\right\rangle = \frac{2\hbar^2}{a}\int_0^a \sin(n\pi x/a)(n\pi/a)^2\sin(n\pi x/a)dx = \frac{n^2\pi^2\hbar^2}{a^2} \tag{0.7}$$

$$\sigma_p^2 = \left\langle p^2 \right\rangle - \left\langle p \right\rangle^2 = \frac{n^2\pi^2\hbar^2}{a^2} \tag{0.8}$$

The uncertainty principle here is then:

$$\sigma_x\sigma_p = \hbar\sqrt{\frac{\pi^2 n^2 - 6}{12}} \tag{0.9}$$

The smallest uncertainty will be for the state $n = 1$ and is approximately $0.568\hbar$, which satisfies the condition $\sigma_x\sigma_p \geq \hbar/2$.

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