

INFINITE SQUARE WELL - UNCERTAINTY PRINCIPLE

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.4.

We can calculate the mean values of position and momentum and verify the uncertainty principle for the infinite square well. The Schrödinger equation for the square well, between $x = 0$ and $x = a$:

$$(1) \quad \frac{d^2\psi}{dx^2} = -\frac{2m}{\hbar^2}E\psi$$

The stationary states of the infinite square well are given by

$$(2) \quad \sqrt{\frac{2}{a}} \sin\left(\frac{n\pi}{a}x\right) e^{-i(n^2\pi^2\hbar/2ma^2)t}$$

for $0 \leq x \leq a$.

For x we have

$$(3) \quad \langle x \rangle = \frac{2}{a} \int_0^a x \sin^2(n\pi x/a) dx = a/2$$

$$(4) \quad \langle x^2 \rangle = \frac{2}{a} \int_0^a x^2 \sin^2(n\pi x/a) dx = a^2 \left(\frac{1}{3} - \frac{1}{2n^2\pi^2} \right)$$

$$(5) \quad \sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2 = a^2 \left(\frac{1}{12} - \frac{1}{2n^2\pi^2} \right) = \frac{n^2\pi^2 - 6}{12n^2\pi^2} a^2$$

For the momentum p we have

$$(6) \quad \langle p \rangle = \frac{2\hbar}{ai} \int_0^a \sin(n\pi x/a) (n\pi/a) \cos(n\pi x/a) dx = 0$$

$$(7) \quad \langle p^2 \rangle = \frac{2\hbar^2}{a} \int_0^a \sin(n\pi x/a) (n\pi/a)^2 \sin(n\pi x/a) dx = \frac{n^2\pi^2\hbar^2}{a^2}$$

$$(8) \quad \sigma_p^2 = \langle p^2 \rangle - \langle p \rangle^2 = \frac{n^2 \pi^2 \hbar^2}{a^2}$$

The uncertainty principle here is then:

$$(9) \quad \sigma_x \sigma_p = \hbar \sqrt{\frac{\pi^2 n^2 - 6}{12}}$$

The smallest uncertainty will be for the state $n = 1$ and is approximately $0.568\hbar$, which satisfies the condition $\sigma_x \sigma_p \geq \hbar/2$.

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