

INFINITE SQUARE WELL - COMBINATION OF TWO LOWEST STATES

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.5.

As an example of an explicit case of a particle in the infinite square well, suppose we have a particle that starts off in a combination of the two lowest states:

$$\Psi(x, 0) = A[\psi_1(x) + \psi_2(x)] \quad (1)$$

To normalize, we find A by using the orthonormal property of the stationary states, so:

$$\int |\Psi(x, 0)|^2 dx = A^2 \int (\psi_1^* + \psi_2^*)(\psi_1 + \psi_2) dx \quad (2)$$

$$= A^2 \int (|\psi_1|^2 + |\psi_2|^2) dx \quad (3)$$

$$= 2A^2 \quad (4)$$

$$= 1 \quad (5)$$

So $A = 1/\sqrt{2}$.

Using $\omega \equiv \pi^2 \hbar / 2ma^2$, we have for the full wave function:

$$\Psi(x, t) = \frac{\sqrt{2}}{2} \psi_1(x) e^{-i\omega t} + \frac{\sqrt{2}}{2} \psi_2(x) e^{-4i\omega t} \quad (6)$$

and

$$\begin{aligned} 2|\Psi(x, t)|^2 &= (\psi_1^*(x)e^{i\omega t} + \psi_2^*(x)e^{4i\omega t})(\psi_1(x)e^{-i\omega t} + \psi_2(x)e^{-4i\omega t}) \quad (7) \\ &= \psi_1^2 + \psi_2^2 + 2\psi_1\psi_2 \cos(3\omega t) \quad (8) \end{aligned}$$

$$|\Psi(x, t)|^2 = \frac{1}{2}(\psi_1^2 + \psi_2^2 + 2\psi_1\psi_2 \cos(3\omega t)) \quad (9)$$

The stationary states are real functions, so we can drop the * notation on the complex conjugates. Note that $\int |\Psi(x,t)|^2 dx = 1$ for all times, since the cosine term integrates to zero due to orthogonality.

The average position is:

$$\langle x \rangle = \frac{1}{2} \int_0^a (x\psi_1^2(x) + x\psi_2^2(x) + 2x\psi_1\psi_2 \cos(3\omega t)) dx \quad (10)$$

$$= \frac{a}{2} - \frac{16a}{9\pi^2} \cos(3\omega t) \quad (11)$$

The particle's mean position oscillates about the midpoint of the well with an amplitude of $16a/9\pi^2 \approx 0.18a$.

The mean momentum can be found the quick way by taking the derivative of $\langle x \rangle$.

$$\langle p \rangle = m \frac{d\langle x \rangle}{dt} = \frac{8\hbar}{3a} \sin(3\omega t) \quad (12)$$

where we have used the definition of ω . Doing it the long way using integration does give the same answer, as can be checked using Maple (or by hand).

The two possible energies are E_1 and E_2 and since the wave function consists of equal contributions from the corresponding stationary states, they occur with equal probability. Thus

$$\langle H \rangle = \frac{1}{2}(E_1 + E_2) = \frac{5\pi^2\hbar^2}{4ma^2} \quad (13)$$

Again, this can be obtained the long way through integration.

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