INFINITE SQUARE WELL - PHASE DIFFERENCE

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.6.

As another example of an explicit case of a particle in the infinite square well, suppose we modify the example in the last post so that the second stationary state has a constant phase relative to the first. That is:

$$\Psi(x,0)=A[\psi_1(x)+e^{i\phi}\psi_2(x)] \tag{1}$$

we can first normalize Ψ by noting that A is the same as in the last example since ψ_1 and ψ_2 are orthogonal. So

$$A=\frac{\sqrt{2}}{2} \tag{2}$$

The time-dependent form is then:

$$\Psi(x,t)=\frac{\sqrt{2}}{2}\left(\psi_1(x)e^{-i\omega t}+\psi_2(x)e^{i(\phi-4\omega t)}\right) \tag{3}$$

where $\psi_n(x)$ is given by the infinite square well formula.

The modulus is calculated in the usual way

$$|\Psi(x,t)|^2 = \Psi^*\Psi \tag{4}$$

$$= \frac{1}{2}[\psi_1^2(x)+\psi_2^2(x)+2\psi_1(x)\psi_2(x)\cos(\phi-3\omega t)] \tag{5}$$

The mean position $\langle x\rangle$ is calculated by integrating this expression multiplied by x:

$$\langle x\rangle = \frac{1}{2}\int_0^a x[\psi_1^2(x)+\psi_2^2(x)+2\psi_1(x)\psi_2(x)\cos(\phi-3\omega t)]dx \tag{6}$$

$$= \frac{a}{2}-\frac{16a}{9\pi^2}\cos(\phi-3\omega t) \tag{7}$$

where we have used the infinite square well formula to substitute for the ψ_n functions, and done the integral with Maple. This reduces to the answer from before when $\phi = 0$.

For $\phi = \pi/2$ we have

$$\langle x\rangle = \frac{a}{2} - \frac{16a}{9\pi^2}\sin(3\omega t) \tag{8}$$

and for $\phi = \pi$we have

$$\langle x\rangle = \frac{a}{2} + \frac{16a}{9\pi^2}\cos(3\omega t) \tag{9}$$

Thus the phase ϕ has no effect on the amplitude of the oscillation; but it shifts the oscillation in time.