

INFINITE SQUARE WELL - PHASE DIFFERENCE

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.6.

As another example of an explicit case of a particle in the infinite square well, suppose we modify the example in the last post so that the second stationary state has a constant phase relative to the first. That is:

$$\Psi(x, 0) = A[\psi_1(x) + e^{i\phi} \psi_2(x)] \quad (1)$$

we can first normalize Ψ by noting that A is the same as in the last example since ψ_1 and ψ_2 are orthogonal. So

$$A = \frac{\sqrt{2}}{2} \quad (2)$$

The time-dependent form is then:

$$\Psi(x, t) = \frac{\sqrt{2}}{2} (\psi_1(x)e^{-i\omega t} + \psi_2(x)e^{i(\phi - 4\omega t)}) \quad (3)$$

where $\psi_n(x)$ is given by the infinite square well formula.

The modulus is calculated in the usual way

$$|\Psi(x, t)|^2 = \Psi^* \Psi \quad (4)$$

$$= \frac{1}{2} [\psi_1^2(x) + \psi_2^2(x) + 2\psi_1(x)\psi_2(x)\cos(\phi - 3\omega t)] \quad (5)$$

The mean position $\langle x \rangle$ is calculated by integrating this expression multiplied by x :

$$\langle x \rangle = \frac{1}{2} \int_0^a x [\psi_1^2(x) + \psi_2^2(x) + 2\psi_1(x)\psi_2(x)\cos(\phi - 3\omega t)] dx \quad (6)$$

$$= \frac{a}{2} - \frac{16a}{9\pi^2} \cos(\phi - 3\omega t) \quad (7)$$

where we have used the infinite square well formula to substitute for the

ψ_n functions, and done the integral with Maple. This reduces to the answer from before when $\phi = 0$.

For $\phi = \pi/2$ we have

$$\langle x \rangle = \frac{a}{2} - \frac{16a}{9\pi^2} \sin(3\omega t) \quad (8)$$

and for $\phi = \pi$ we have

$$\langle x \rangle = \frac{a}{2} + \frac{16a}{9\pi^2} \cos(3\omega t) \quad (9)$$

Thus the phase ϕ has no effect on the amplitude of the oscillation; but it shifts the oscillation in time.