

INFINITE SQUARE WELL - PARTICLE IN LEFT HALF

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.8.

As another example of an explicit case of a particle in the infinite square well, we can consider a particle that starts off in a state where it is equally likely to be found anywhere in the left half of the well. This means that the wave function is:

$$(1) \quad \Psi(x,0) = \begin{cases} A & 0 < x \leq \frac{a}{2} \\ 0 & \frac{a}{2} < x \leq a \end{cases}$$

and zero everywhere else.

Normalizing, we require

$$(2) \quad \int_0^{a/2} A^2 dx = 1$$

so

$$(3) \quad A = \sqrt{\frac{2}{a}}$$

To find the probability that the particle is in the ground state (with energy $\pi^2 \hbar^2 / 2ma^2$), we need to find the coefficient c_1 in the expansion of $\Psi(x,0)$ in terms of the orthonormal function set. Thus:

$$(4) \quad c_1 = \int_0^a \Psi(x,0) \psi_1(x) dx$$

$$(5) \quad = \frac{2}{a} \int_0^{a/2} \sin(\pi x/a) dx$$

$$(6) \quad = \frac{2}{\pi}$$

The probability of this energy is then $c_1^2 = 4/\pi^2 \approx 0.405285$.