INFINITE SQUARE WELL - PARTICLE IN LEFT HALF

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.8.

As another example of an explicit case of a particle in the infinite square well, we can consider a particle that starts off in a state where it is equally likely to be found anywhere in the left half of the well. This means that the wave function is:

$$\Psi(x,0) = \begin{cases} A & 0 < x \leq \frac{a}{2} \\ 0 & \frac{a}{2} < x \leq a \end{cases} \tag{1}$$

and zero everywhere else.

Normalizing, we require

$$\int_0^{a/2} A^2 dx = 1 \tag{2}$$

so

$$A = \sqrt{\frac{2}{a}} \tag{3}$$

To find the probability that the particle is in the ground state (with energy $\pi^2\hbar^2/2ma^2$), we need to find the coefficient c_1 in the expansion of $\Psi(x,0)$ in terms of the orthonormal function set. Thus:

$$\begin{aligned} c_1 &= \int_0^a \Psi(x,0)\psi_1(x)dx & (4) \\ &= \frac{2}{a}\int_0^{a/2} \sin(\pi x/a)dx & (5) \\ &= \frac{2}{\pi} & (6) \end{aligned}$$

The probability of this energy is then $c_1^2 = 4/\pi^2 \approx 0.405285$.