

INFINITE SQUARE WELL - AVERAGE ENERGY

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.9.

The average or expectation value of the energy of a particle in an infinite square well can be worked out either by using the series solution in the form

$$\langle H \rangle = \sum_n |c_n|^2 E_n \quad (1)$$

or directly using an integral, using $H = p^2/2m$ and $p = (\hbar/i)(d/dx)$:

$$\langle H \rangle = -\frac{\hbar^2}{2m} \int_0^a \Psi^*(x,t) \frac{d^2}{dx^2} \Psi(x,t) dx \quad (2)$$

Since $\Psi(x,t)$ in the general case is a sum over stationary states, as in

$$\Psi(x,t) = \sum_n c_n \psi_n(x) e^{-iE_n t/\hbar} \quad (3)$$

and the $\psi_n(x)$ functions are orthogonal, all the terms of the form

$$-\frac{\hbar^2}{2m} \int_0^a c_n^* \psi_n^*(x) e^{iE_n t/\hbar} c_m \psi_m(x) e^{-iE_m t/\hbar} dx \quad (4)$$

where $m \neq n$ in the integral evaluate to zero, and the remaining terms where $n = m$ are all independent of time since the complex exponentials cancel out, so $\langle H \rangle$ is independent of time. Thus if we know the wave function at any given time, we can use it to work out $\langle H \rangle$ at all times.

For example, if we have a parabolic wave function at $t = 0$

$$\Psi(x,0) = Ax(a-x) \quad (5)$$

for $0 \leq x \leq a$, we can work out $\langle H \rangle$ directly from it. Applying the normalization condition we can find A .

$$|A|^2 \int_0^a x^2 (a-x)^2 dx = 1 \quad (6)$$

$$A = \sqrt{\frac{30}{a^5}} \quad (7)$$

We get for $\langle H \rangle$:

$$\langle H \rangle = \frac{30}{a^5} \int_0^a x(a-x) \left(-\frac{\hbar^2}{2m} \frac{d^2}{dx^2} x(a-x) \right) dx \quad (8)$$

Working out the second derivative and then the integral (use Maple or do by hand) we get $\langle H \rangle = 5\hbar^2/ma^2$.