

HARMONIC OSCILLATOR - CHANGE IN SPRING CONSTANT

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.14.

Suppose we have a harmonic oscillator that starts off in the ground state, with frequency ω . If nothing changes in this system, the oscillator will remain in the ground state. However, if we quadruple the spring constant k , this will cause a change in the frequency, since $k = m\omega^2$. Thus ω will double to a new value $\omega' = 2\omega$. In terms of this new frequency, we have

$$\begin{aligned} (0.1) \quad \Psi(x, 0) &= \psi_0(x) \\ (0.2) \quad &= \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar} \\ (0.3) \quad &= \left(\frac{m\omega'}{2\pi\hbar}\right)^{1/4} e^{-m\omega' x^2/4\hbar} \end{aligned}$$

After the change in the spring constant, the new stationary states will be the functions ψ'_0, ψ'_1, \dots which have the same form as the original ψ_n functions except ω is replaced by ω' . Thus the new ground state is

$$(0.4) \quad \psi'_0(x) = \left(\frac{m\omega'}{\pi\hbar}\right)^{1/4} e^{-m\omega' x^2/2\hbar}$$

The energy of this state is $\hbar\omega'/2 = \hbar\omega$. Since this is the new ground state, the probability of finding the energy at the old value of $\hbar\omega/2$ is now zero, since the energy cannot be less than the ground state energy.

To find the probability that the energy is the ground state energy of $\hbar\omega$, we expand the original wave function in terms of the new stationary states and find the coefficient c_0 of the first term in the expansion. That is

$$(0.5) \quad \Psi(x, 0) = \sum_{n=0}^{\infty} c_n \psi'_n(x)$$

Since the ψ'_n are orthogonal functions, we can use the usual method of calculating c_0 :

$$(0.6) \quad c_0 = \int_0^a \Psi(x, 0) \psi'_0 dx$$

$$(0.7) \quad = \left(\frac{m\omega'}{2\pi\hbar} \right)^{1/4} \left(\frac{m\omega'}{\pi\hbar} \right)^{1/4} \int_{-\infty}^{\infty} e^{-m\omega'x^2/4\hbar} e^{-m\omega'x^2/2\hbar} dx$$

$$(0.8) \quad = \frac{1}{2^{1/4}} \left(\frac{m\omega'}{\pi\hbar} \right)^{1/2} \int_{-\infty}^{\infty} e^{-3m\omega'x^2/4\hbar} dx$$

$$(0.9) \quad = \frac{1}{2^{1/4}} \left(\frac{m\omega'}{\pi\hbar} \right)^{1/2} \frac{2}{3} \left(\frac{3\pi\hbar}{m\omega'} \right)^{1/2}$$

$$(0.10) \quad = \frac{2^{3/4}}{3^{1/2}}$$

The probability of being in the ground state is thus

$$(0.11) \quad |c_0|^2 = \frac{2^{3/2}}{3}$$

$$(0.12) \quad = 0.9428$$