HARMONIC OSCILLATOR - CHANGE IN SPRING CONSTANT

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.14.

Suppose we have a harmonic oscillator that starts off in the ground state, with frequency ω. If nothing changes in this system, the oscillator will remain in the ground state. However, if we quadruple the spring constant k, this will cause a change in the frequency, since $k = m\omega^2$. Thus ω will double to a new value $\omega' = 2\omega$. In terms of this new frequency, we have

$$\Psi(x,0) = \psi_0(x) \tag{1}$$

$$= \left(\frac{m\omega}{\pi\hbar}\right)^{1/4} e^{-m\omega x^2/2\hbar} \tag{2}$$

$$= \left(\frac{m\omega'}{2\pi\hbar}\right)^{1/4} e^{-m\omega' x^2/4\hbar} \tag{3}$$

After the change in the spring constant, the new stationary states will be the functions ψ_0', $\psi_1', \ldots$ which have the same form as the original ψ_n functions except ω is replaced by ω'. Thus the new ground state is

$$\psi_0'(x) = \left(\frac{m\omega'}{\pi\hbar}\right)^{1/4} e^{-m\omega' x^2/2\hbar} \tag{4}$$

The energy of this state is $\hbar\omega'/2 = \hbar\omega$. Since this is the new ground state, the probability of finding the energy at the old value of $\hbar\omega/2$ is now zero, since the energy cannot be less than the ground state energy.

To find the probability that the energy is the ground state energy of $\hbar\omega$, we expand the original wave function in terms of the new stationary states and find the coefficient c_0 of the first term in the expansion. That is

$$\Psi(x,0) = \sum_{n=0}^{\infty} c_n \psi_n'(x) \tag{5}$$

Since the ψ_n' are orthogonal functions, we can use the usual method of calculating c_0:

$$c_0 = \int_0^a \Psi(x,0)\psi_0' dx \tag{6}$$

$$= \left(\frac{m\omega'}{2\pi\hbar}\right)^{1/4}\left(\frac{m\omega'}{\pi\hbar}\right)^{1/4}\int_{-\infty}^{\infty} e^{-m\omega' x^2/4\hbar} e^{-m\omega' x^2/2\hbar} dx \tag{7}$$

$$= \frac{1}{2^{1/4}}\left(\frac{m\omega'}{\pi\hbar}\right)^{1/2}\int_{-\infty}^{\infty} e^{-3m\omega' x^2/4\hbar} dx \tag{8}$$

$$= \frac{1}{2^{1/4}}\left(\frac{m\omega'}{\pi\hbar}\right)^{1/2}\frac{2}{3}\left(\frac{3\pi\hbar}{m\omega'}\right)^{1/2} \tag{9}$$

$$= \frac{2^{3/4}}{3^{1/2}} \tag{10}$$

The probability of being in the ground state is thus

$$|c_0|^2 = \frac{2^{3/2}}{3} \tag{11}$$

$$= 0.9428 \tag{12}$$