COMPLEX EXPONENTIALS AND TRIG FUNCTIONS

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.18.

Solutions to the Schrödinger equation are written either as complex exponentials or a sum of sine and cosine functions. For example, we used the former in the solution for the free particle and the latter in the infinite square well. In fact these two ways are equivalent for writing a function, as we can demonstrate.

We start with

$$Ae^{ikx} + Be^{-ikx} \tag{1}$$

and we want to write this as $C\cos kx + D\sin kx$.

We can expand the exponentials in terms of sine and cosine to get

$$\begin{aligned} Ae^{ikx} + Be^{-ikx} &= (A+B)\cos kx + i(A-B)\sin x && (2)\\ &\equiv C\cos kx + D\sin kx && (3) \end{aligned}$$

So

$$\begin{aligned} C &= A+B && (4)\\ D &= i(A-B) && (5) \end{aligned}$$

or, the other way round:

$$\begin{aligned} A &= \frac{1}{2}(C-iD) && (6)\\ B &= \frac{1}{2}(C+iD) && (7) \end{aligned}$$

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