

THE FREE PARTICLE: PROBABILITY CURRENT

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.19.

The rate of change of probability of a particle in a given range of x can be written as the difference in probability current at the two ends. The current is defined as

$$(0.1) \quad J(x,t) \equiv \frac{i\hbar}{2m} \left(\frac{\partial\Psi^*}{\partial x} \Psi - \frac{\partial\Psi}{\partial x} \Psi^* \right)$$

For the free particle, a stationary state is given by

$$(0.2) \quad \Psi(x,t) = Ae^{ikx} e^{-i\hbar k^2 t/2m}$$

The probability current for this state is found by working out the derivative:

$$(0.3) \quad \frac{\partial\Psi}{\partial x} = ikAe^{ikx} e^{-i\hbar k^2 t/2m}$$

$$(0.4) \quad = ik\Psi$$

So we get

$$(0.5) \quad J(x,t) = \frac{i\hbar k}{2m} \left(-i|\Psi|^2 - i|\Psi|^2 \right)$$

$$(0.6) \quad = \frac{\hbar k}{m} |A|^2$$

(The complex exponentials cancel out in $|\Psi|^2$.) Since the current is positive, it 'flows' in the positive x direction. Note that the current is independent of x , so the probability of a particle being found in any given range of x is constant. (Actually, as we've seen, a free particle can't exist in a single stationary state since such a state cannot be normalized.)

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