

## THE FREE PARTICLE: PROBABILITY CURRENT

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.19.

The rate of change of probability of a particle in a given range of  $x$  can be written as the difference in probability current at the two ends. The current is defined as

$$J(x,t) \equiv \frac{i\hbar}{2m} \left( \frac{\partial\Psi^*}{\partial x} \Psi - \frac{\partial\Psi}{\partial x} \Psi^* \right) \quad (1)$$

For the free particle, a stationary state is given by

$$\Psi(x,t) = Ae^{ikx} e^{-i\hbar k^2 t/2m} \quad (2)$$

The probability current for this state is found by working out the derivative:

$$\frac{\partial\Psi}{\partial x} = ikAe^{ikx} e^{-i\hbar k^2 t/2m} \quad (3)$$

$$= ik\Psi \quad (4)$$

So we get

$$J(x,t) = \frac{i\hbar k}{2m} \left( -i|\Psi|^2 - i|\Psi|^2 \right) \quad (5)$$

$$= \frac{\hbar k}{m} |A|^2 \quad (6)$$

(The complex exponentials cancel out in  $|\Psi|^2$ .) Since the current is positive, it 'flows' in the positive  $x$  direction. Note that the current is independent of  $x$ , so the probability of a particle being found in any given range of  $x$  is constant. (Actually, as we've seen, a free particle can't exist in a single stationary state since such a state cannot be normalized.)

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