THE FREE PARTICLE: PROBABILITY CURRENT

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.19.

The rate of change of probability of a particle in a given range of x can be written as the difference in probability current at the two ends. The current is defined as

$$J(x,t) \equiv \frac{i\hbar}{2m}\left(\frac{\partial\Psi^*}{\partial x}\Psi - \frac{\partial\Psi}{\partial x}\Psi^*\right) \tag{1}$$

For the free particle, a stationary state is given by

$$\Psi(x,t) = Ae^{ikx}e^{-i\hbar k^2 t/2m} \tag{2}$$

The probability current for this state is found by working out the derivative:

$$\begin{aligned}\frac{\partial\Psi}{\partial x} &= ikAe^{ikx}e^{-i\hbar k^2 t/2m} &(3)\\ &= ik\Psi &(4)\end{aligned}$$

So we get

$$\begin{aligned}J(x,t) &= \frac{i\hbar k}{2m}\left(-i|\Psi|^2 - i|\Psi|^2\right) &(5)\\ &= \frac{\hbar k}{m}|A|^2 &(6)\end{aligned}$$

(The complex exponentials cancel out in $|\Psi|^2$.) Since the current is positive, it 'flows' in the positive x direction. Note that the current is independent of x, so the probability of a particle being found in any given range of x is constant. (Actually, as we've seen, a free particle can't exist in a single stationary state since such a state cannot be normalized.)

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