

## DIRAC DELTA FUNCTION - SIMPLE EXAMPLES

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Post date: 30 Jul 2012.

Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.23.

Here are a few simple examples of integrals involving the Dirac delta function. The delta function is defined by the two conditions:

$$(1) \quad \delta(x) = 0 \text{ if } x \neq 0$$
$$(2) \quad \int_{-\infty}^{\infty} \delta(x) dx = 1$$

Since it is zero everywhere except at  $x = 0$  it follows that

$$(3) \quad \int_{-\infty}^{\infty} f(x) \delta(x) dx = f(0)$$

for any 'ordinary' function  $f(x)$ . A simple extension of this is

$$(4) \quad \int_a^b f(x) \delta(x-k) dx = \begin{cases} f(k) & a < k < b \\ 0 & \text{otherwise} \end{cases}$$

This follows by making the substitution  $y = x - k$ . Then  $dx = dy$  and we get

$$(5) \quad \int_a^b f(x) \delta(x-k) dx = \int_{a-k}^{b-k} f(y+k) \delta(y) dy$$

This integral is  $f(y=0) = f(k)$  provided the limits of integration include  $y = 0$ , that is,  $a - k < 0 < b - k$ , or  $a < k < b$ .

For example

$$(6) \quad \int_{-3}^1 f(x) \delta(x+2) dx = \int_{-3}^1 (x^3 - 3x^2 + 2x - 1) \delta(x+2) dx$$

$$(7) \quad = f(-2)$$

$$(8) \quad = -25$$

Another example:

$$(9) \quad \int_0^{\infty} f(x)\delta(x-\pi)dx = \int_0^{\infty} [\cos(3x)+2]\delta(x-\pi)dx$$

$$(10) \quad = f(\pi)$$

$$(11) \quad = 1$$

And a final example:

$$(12) \quad \int_{-1}^1 e^{|x|+3}\delta(x-2)dx = 0$$

since the limits of integration don't include  $x = 2$ .