

DIRAC DELTA FUNCTION - SIMPLE EXAMPLES

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.23.

Here are a few simple examples of integrals involving the Dirac delta function. The delta function is defined by the two conditions:

$$\delta(x) = 0 \text{ if } x \neq 0 \quad (1)$$

$$\int_{-\infty}^{\infty} \delta(x) dx = 1 \quad (2)$$

Since it is zero everywhere except at $x = 0$ it follows that

$$\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(0) \quad (3)$$

for any 'ordinary' function $f(x)$. A simple extension of this is

$$\int_a^b f(x)\delta(x-k)dx = \begin{cases} f(k) & a < k < b \\ 0 & \text{otherwise} \end{cases} \quad (4)$$

This follows by making the substitution $y = x - k$. Then $dx = dy$ and we get

$$\int_a^b f(x)\delta(x-k)dx = \int_{a-k}^{b-k} f(y+k)\delta(y)dy \quad (5)$$

This integral is $f(y=0) = f(k)$ provided the limits of integration include $y = 0$, that is, $a - k < 0 < b - k$, or $a < k < b$.

For example

$$\int_{-3}^1 f(x)\delta(x+2)dx = \int_{-3}^1 (x^3 - 3x^2 + 2x - 1)\delta(x+2)dx \quad (6)$$

$$= f(-2) \quad (7)$$

$$= -25 \quad (8)$$

Another example:

$$\int_0^{\infty} f(x) \delta(x - \pi) dx = \int_0^{\infty} [\cos(3x) + 2] \delta(x - \pi) dx \quad (9)$$

$$= f(\pi) \quad (10)$$

$$= 1 \quad (11)$$

And a final example:

$$\int_{-1}^1 e^{|x|+3} \delta(x - 2) dx = 0 \quad (12)$$

since the limits of integration don't include $x = 2$.