DIRAC DELTA FUNCTION - SIMPLE EXAMPLES

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.23.

Here are a few simple examples of integrals involving the Dirac delta function. The delta function is defined by the two conditions:

$$\begin{aligned} \delta(x) &= 0 \text{ if } x \neq 0 && (1) \\ \int_{-\infty}^{\infty} \delta(x)dx &= 1 && (2) \end{aligned}$$

Since it is zero everywhere except at $x = 0$ it follows that

$$\int_{-\infty}^{\infty} f(x)\delta(x)dx = f(0) \tag{3}$$

for any 'ordinary' function $f(x)$. A simple extension of this is

$$\int_a^b f(x)\delta(x-k)dx = \begin{cases} f(k) & a < k < b \\ 0 & \text{otherwise} \end{cases} \tag{4}$$

This follows by making the substitution $y = x - k$. Then $dx = dy$ and we get

$$\int_a^b f(x)\delta(x-k)dx = \int_{a-k}^{b-k} f(y+k)\delta(y)dy \tag{5}$$

This integral is $f(y = 0) = f(k)$ provided the limits of integration include $y = 0$, that is, $a - k < 0 < b - k$, or $a < k < b$.

For example

$$\begin{aligned} \int_{-3}^{1} f(x)\delta(x+2)dx &= \int_{-3}^{1} \left(x^3 - 3x^2 + 2x - 1\right)\delta(x+2)dx && (6) \\ &= f(-2) && (7) \\ &= -25 && (8) \end{aligned}$$

Another example:

$$\int_0^\infty f(x)\delta(x-\pi)dx = \int_0^\infty [\cos(3x)+2]\,\delta(x-\pi)dx \quad (9)$$
$$= f(\pi) \quad (10)$$
$$= 1 \quad (11)$$

And a final example:

$$\int_{-1}^{1} e^{|x|+3}\delta(x-2)dx = 0 \quad (12)$$

since the limits of integration don't include $x = 2$.