FINITE DROP POTENTIAL

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.35.

The problem of the finite step potential can be inverted to give a finite drop potential by replacing V_0 by $-V_0$, so the potential is given by

$$V(x)=\begin{cases}0 & x\le 0\\ -V_0 & x>0\end{cases} \tag{0.1}$$

If we assume particles coming in from the left, then we must have $E>0$ (otherwise the wave function would decay exponentially inside the barrier and we couldn't have particles coming in from infinity on the left). In this case the reflection coefficient is

$$R=\left[\frac{E-\sqrt{E(E+V_0)}}{E+\sqrt{E(E+V_0)}}\right]^2 \tag{0.2}$$

and the transmission coefficient is

$$T=\frac{4E^{3/2}\sqrt{E+V_0}}{\left(E+\sqrt{E(E+V_0)}\right)^2} \tag{0.3}$$

For $V_0=0$ the problem reduces to that of the free particle, and $R=0$, $T=1$ as we'd expect. As V_0 gets very large, $R\to 1$, $T\to 0$.

If we take $E=V_0/3$, then $R=1/9$.

Although the graph of the potential looks like a cliff, it doesn't represent the behaviour of an object, such as a car, falling over a cliff. Classically, the energy of a car is kinetic + potential, which in the absence of other forces, remains a constant. If a car had a speed v_1 in a region where $V=0$, then its total energy is kinetic: $E=K_1=\frac{1}{2}mv_1^2$. If it suddenly encounters a region where $V=-V_0$, then we'd have $E=K_2-V_0$, so K_2 is larger than K_1, meaning that the car would instantaneously increase its speed, which of course doesn't happen. In reality, a car driving off a cliff encounters a potential energy of $-mgy$ where y is the distance it has fallen, so its kinetic energy increases gradually. Besides, a car falling off a cliff is essentially a two-dimensional problem, so trying to analyze it in one dimension won't work.

A slightly more realistic case is that of a neutron which is fired at an atomic nucleus. The neutron experiences a sudden drop in potential from $V = 0$ outside the nucleus to $V = -V_0 = -12$ MeV inside. If we give the neutron an initial kinetic energy of $E = 4$ MeV, then the probability of transmission into the nucleus is

$$T = \frac{4 \times 4^{3/2}\sqrt{4+12}}{\left(4+\sqrt{4(4+12)}\right)^2} \tag{0.4}$$

$$= \frac{128}{144} \tag{0.5}$$

$$= \frac{8}{9} \tag{0.6}$$