

FINITE DROP POTENTIAL

Link to: [physicspages home page](#).

To leave a comment or report an error, please use the auxiliary blog.

Post date: 9 Aug 2012.

Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.35.

The problem of the finite step potential can be inverted to give a finite drop potential by replacing V_0 by $-V_0$, so the potential is given by

$$V(x) = \begin{cases} 0 & x \leq 0 \\ -V_0 & x > 0 \end{cases} \quad (1)$$

If we assume particles coming in from the left, then we must have $E > 0$ (otherwise the wave function would decay exponentially inside the barrier and we couldn't have particles coming in from infinity on the left). In this case the reflection coefficient is

$$R = \left[\frac{E - \sqrt{E(E + V_0)}}{E + \sqrt{E(E + V_0)}} \right]^2 \quad (2)$$

and the transmission coefficient is

$$T = \frac{4E^{3/2}\sqrt{E + V_0}}{\left(E + \sqrt{E(E + V_0)}\right)^2} \quad (3)$$

For $V_0 = 0$ the problem reduces to that of the free particle, and $R = 0$, $T = 1$ as we'd expect. As V_0 gets very large, $R \rightarrow 1$, $T \rightarrow 0$.

If we take $E = V_0/3$, then $R = 1/9$.

Although the graph of the potential looks like a cliff, it doesn't represent the behaviour of an object, such as a car, falling over a cliff. Classically, the energy of a car is kinetic + potential, which in the absence of other forces, remains a constant. If a car had a speed v_1 in a region where $V = 0$, then its total energy is kinetic: $E = K_1 = \frac{1}{2}mv_1^2$. If it suddenly encounters a region where $V = -V_0$, then we'd have $E = K_2 - V_0$, so K_2 is larger than K_1 , meaning that the car would instantaneously increase its speed, which of course doesn't happen. In reality, a car driving off a cliff encounters a potential energy of $-mgy$ where y is the distance it has fallen, so its kinetic energy increases gradually. Besides, a car falling off a cliff is essentially a

two-dimensional problem, so trying to analyze it in one dimension won't work.

A slightly more realistic case is that of a neutron which is fired at an atomic nucleus. The neutron experiences a sudden drop in potential from $V = 0$ outside the nucleus to $V = -V_0 = -12$ MeV inside. If we give the neutron an initial kinetic energy of $E = 4$ MeV, then the probability of transmission into the nucleus is

$$T = \frac{4 \times 4^{3/2} \sqrt{4+12}}{\left(4 + \sqrt{4(4+12)}\right)^2} \quad (4)$$

$$= \frac{128}{144} \quad (5)$$

$$= \frac{8}{9} \quad (6)$$