FINITE DROP POTENTIAL

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.35.

The problem of the finite step potential can be inverted to give a finite drop potential by replacing V_0 by $-V_0$, so the potential is given by

$$V(x) = \begin{cases} 0 & x \le 0 \\ -V_0 & x > 0 \end{cases}$$
(1)

If we assume particles coming in from the left, then we must have E > 0 (otherwise the wave function would decay exponentially inside the barrier and we couldn't have particles coming in from infinity on the left). In this case the reflection coefficient is

$$R = \left[\frac{E - \sqrt{E(E + V_0)}}{E + \sqrt{E(E + V_0)}}\right]^2 \tag{2}$$

and the transmission coefficient is

$$T = \frac{4E^{3/2}\sqrt{E+V_0}}{\left(E+\sqrt{E(E+V_0)}\right)^2}$$
(3)

For $V_0 = 0$ the problem reduces to that of the free particle, and R = 0, T = 1 as we'd expect. As V_0 gets very large, $R \to 1, T \to 0$. If we take E = V/2 then R = 1/0

If we take $E = V_0/3$, then R = 1/9.

Although the graph of the potential looks like a cliff, it doesn't represent the behaviour of an object, such as a car, falling over a cliff. Classically, the energy of a car is kinetic + potential, which in the absence of other forces, remains a constant. If a car had a speed v_1 in a region where V = 0, then its total energy is kinetic: $E = K_1 = \frac{1}{2}mv_1^2$. If it suddenly encounters a region where $V = -V_0$, then we'd have $E = K_2 - V_0$, so K_2 is larger than K_1 , meaning that the car would instantaneously increase its speed, which of course doesn't happen. In reality, a car driving off a cliff encounters a potential energy of -mgy where y is the distance it has fallen, so its kinetic energy increases gradually. Besides, a car falling off a cliff is essentially a two-dimensional problem, so trying to analyze it in one dimension won't work.

A slightly more realistic case is that of a neutron which is fired at an atomic nucleus. The neutron experiences a sudden drop in potential from V = 0 outside the nucleus to $V = -V_0 = -12$ MeV inside. If we give the neutron an initial kinetic energy of E = 4 MeV, then the probability of transmission into the nucleus is

$$T = \frac{4 \times 4^{3/2} \sqrt{4 + 12}}{\left(4 + \sqrt{4(4 + 12)}\right)^2} \tag{4}$$

$$=\frac{128}{144}$$
 (5)

$$= \frac{8}{9} \tag{6}$$