FINITE DROP POTENTIAL

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Post date: 9 Aug 2012.

The problem of the finite step potential can be inverted to give a finite drop potential by replacing $V_0$ by $-V_0$, so the potential is given by

$$V(x) = \begin{cases} 0 & x \leq 0 \\ -V_0 & x > 0 \end{cases}$$

(1)

If we assume particles coming in from the left, then we must have $E > 0$ (otherwise the wave function would decay exponentially inside the barrier and we couldn’t have particles coming in from infinity on the left). In this case the reflection coefficient is

$$R = \left[ \frac{E - \sqrt{E(E + V_0)}}{E + \sqrt{E(E + V_0)}} \right]^2$$

(2)

and the transmission coefficient is

$$T = \frac{4E^{3/2}\sqrt{E + V_0}}{(E + \sqrt{E(E + V_0)})^2}$$

(3)

For $V_0 = 0$ the problem reduces to that of the free particle, and $R = 0$, $T = 1$ as we’d expect. As $V_0$ gets very large, $R \to 1$, $T \to 0$.

If we take $E = V_0/3$, then $R = 1/9$.

Although the graph of the potential looks like a cliff, it doesn’t represent the behaviour of an object, such as a car, falling over a cliff. Classically, the energy of a car is kinetic + potential, which in the absence of other forces, remains a constant. If a car had a speed $v_1$ in a region where $V = 0$, then its total energy is kinetic: $E = K_1 = \frac{1}{2}mv_1^2$. If it suddenly encounters a region where $V = -V_0$, then we’d have $E = K_2 - V_0$, so $K_2$ is larger than $K_1$, meaning that the car would instantaneously increase its speed, which of course doesn’t happen. In reality, a car driving off a cliff encounters a potential energy of $-mgy$ where $y$ is the distance it has fallen, so its kinetic energy increases gradually. Besides, a car falling off a cliff is essentially a
two-dimensional problem, so trying to analyze it in one dimension won’t work.

A slightly more realistic case is that of a neutron which is fired at an atomic nucleus. The neutron experiences a sudden drop in potential from $V = 0$ outside the nucleus to $V = -V_0 = -12$ MeV inside. If we give the neutron an initial kinetic energy of $E = 4$ MeV, then the probability of transmission into the nucleus is

$$T = \frac{4 \times 4^{3/2} \sqrt{4 + 12}}{(4 + \sqrt{4(4 + 12)})^2}$$

$$= \frac{128}{144}$$

$$= \frac{8}{9}$$