HALF-HARMONIC OSCILLATOR

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.42.

Shankar, R. (1994), *Principles of Quantum Mechanics*, Plenum Press. Section 7.3, Exercise 7.3.6.

Suppose we modify the harmonic oscillator potential so that it becomes a half-harmonic oscillator. That is

$$V(x)=\begin{cases}\infty & x<0\\ \frac{1}{2}m\omega^2x^2 & x>0\end{cases} \tag{1}$$

A physical interpretation of this could be a spring that can be stretched from its equilibrium position but not compressed.

We can find the allowed energies of this potential by considering its difference from the ordinary harmonic oscillator. In the ordinary case, there were no boundary conditions, and we found that the stationary states could be expressed in terms of the Hermite polynomials

$$\psi_n(x)=\left(\frac{m\omega}{\pi\hbar}\right)^{1/4}\frac{1}{\sqrt{2^n n!}}H_n\left(\sqrt{\frac{m\omega}{\hbar}}x\right)e^{-m\omega x^2/2\hbar} \tag{2}$$

The Hermite polynomials are even if n is even and odd if n is odd. Since all the even Hermite polynomials have a non-zero constant term, $H_n(0)\neq 0$ if n is even. Similarly, since all odd Hermite polynomials have no constant term, $H(0)=0$ if n is odd.

From continuity of the wave function at $x=0$ we must have $\psi(0)=0$ (since the wave function is zero for $x<0$). The solution above still applies for $x>0$, but due to the boundary condition, we are allowed only the odd Hermite polynomial solutions for $x>0$, which in turn means that the allowed energies are

$$E_n=\left(n+\frac{1}{2}\right)\hbar\omega \tag{3}$$

for n odd only.