DEGENERATE SOLUTIONS DON'T EXIST IN ONE DIMENSION

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.45.

If two distinct solutions to the Schrödinger equation (that is, solutions that aren't merely multiples of each other) have the same energy, they are said to be *degenerate*. Although degenerate states do exist in nature (that is, in a three-dimensional world), there are no degenerate solutions to the Schrödinger equation in any one-dimensional system, regardless of the potential.

To see this, assume there are distinct solutions ψ_1 and ψ_2 that have the same energy E. Then the Schrödinger equation for these two wave functions is:

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi_1}{\partial x^2}+V\psi_1 = E\psi_1 \tag{0.1}$$

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi_2}{\partial x^2}+V\psi_2 = E\psi_2 \tag{0.2}$$

Multiply the first equation by ψ_2 and the second by $\psi_1$and subtract to get:

$$\psi_2\psi_1''-\psi_1\psi_2''=0 \tag{0.3}$$

The expression on the left is the derivative of $\psi_1\psi_2'-\psi_2\psi_1'$ so we can integrate it to get:

$$\psi_1\psi_2'-\psi_2\psi_1'=C \tag{0.4}$$

where C is some constant. If the domain of these functions extends to infinity, then the functions and their derivatives must tend to zero at infinity in order for them to be normalizable. Since 0.4 is valid for all x, C must be zero. We can then rearrange 0.4 as:

$$\frac{\psi_2'}{\psi_2}=\frac{\psi_1'}{\psi_1} \tag{0.5}$$

Integrating this leads to $\ln\psi_2 = \ln\psi_1 + K$ for another constant K, so $\psi_2 = e^K\psi_1$ and any two solutions with the same energy must be multiples of each other. Thus there are no distinct degenerate states in one dimension.

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