PARTICLE ON A CIRCULAR WIRE

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.46.

Suppose we have a particle that slides on a frictionless circular loop of circumference L. Since there is no friction, $V = 0$, but unlike the free particle, we have a periodic constraint that $\psi(x+L) = \psi(x)$.

Since $V = 0$, the solution is the same as that for the free particle, with the added condition that $0 < x < L$:

$$\Psi_k(x,t) = Ae^{i(kx-\hbar k^2 t/2m)} \tag{1}$$

Normalizing this by integrating the square modulus over $[0,L]$ gives $A = 1/\sqrt{L}$.

The constraint here is that $\Psi(x) = \Psi(x+L)$, where L is the circumference of the wire. This leads to $e^{i(kx+kL)} = e^{ikx}$, so the condition on k becomes $k = 2n\pi/L$. Since the solution above assumes that k can be positive or negative, n takes on all positive and negative integer values. From this we can get the allowed energies:

$$E_{\pm n} = \frac{2n^2\pi^2\hbar^2}{mL^2} \tag{2}$$

Note that the energy is the same for $\pm n$, so in this case there are two wave functions for each energy meaning that we have a one-dimensional system with degenerate states. However, in the proof that such states cannot exist, we assumed that the wave function was defined over all x, not just a restricted range. Thus the condition that the wave function goes to zero at infinity is not true here, so the conditions for non-degeneracy specified in the proof do not hold here.

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