# PARTICLE ON A CIRCULAR WIRE

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.46.

Suppose we have a particle that slides on a frictionless circular loop of circumference $L$. Since there is no friction, $V=0$, but unlike the free particle, we have a periodic constraint that $\psi(x+L)=\psi(x)$.

Since $V=0$, the solution is the same as that for the free particle, with the added condition that $0<x<L$:

$$\Psi_k(x,t)=Ae^{i(kx-\hbar k^2t/2m)} \tag{1}$$

Normalizing this by integrating the square modulus over $[0,L]$ gives $A=1/\sqrt{L}$.

The constraint here is that $\Psi(x)=\Psi(x+L)$, where $L$ is the circumference of the wire. This leads to $e^{i(kx+kL)}=e^{ikx}$, so the condition on $k$ becomes $k=2n\pi/L$. Since the solution above assumes that $k$ can be positive or negative, $n$ takes on all positive and negative integer values. From this we can get the allowed energies:

$$E_{\pm n}=\frac{2n^2\pi^2\hbar^2}{mL^2} \tag{2}$$

Note that the energy is the same for $\pm n$, so in this case there are two wave functions for each energy meaning that we have a one-dimensional system with degenerate states. However, in the proof that such states cannot exist, we assumed that the wave function was defined over all $x$, not just a restricted range. Thus the condition that the wave function goes to zero at infinity is not true here, so the conditions for non-degeneracy specified in the proof do not hold here.

## PINGBACKS

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