

INFINITE SQUARE WELL WITH TRIANGULAR INITIAL STATE USING DELTA FUNCTION

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.48.

Earlier, we looked at the case of a particle in the infinite square well with an initial wave function that was triangular:

$$(1) \quad \Psi(x, 0) = \begin{cases} Ax & 0 \leq x \leq \frac{a}{2} \\ A(a-x) & \frac{a}{2} \leq x \leq a \end{cases}$$

We find A from normalization:

$$(2) \quad \int_0^a |\Psi|^2 dx = A^2 a^3 / 12 = 1$$

so

$$(3) \quad A = \frac{\sqrt{12}}{a^{3/2}}$$

Because the first derivative of $\Psi(x, 0)$ is discontinuous at $x = 0$, we might encounter problems in calculating the second derivative, which we need to find the mean value of the energy if we use integration, since this is

$$(4) \quad \langle E \rangle = \frac{\hbar^2}{2m} \int_0^a \Psi^*(x, 0) \frac{d^2}{dx^2} \Psi(x, 0) dx$$

We can express the first derivative of wave function as a step function $\theta(x)$:

$$(5) \quad \frac{d\Psi(x, 0)}{dx} = \begin{cases} A & 0 < x < a/2 \\ -A & a/2 < x < a \end{cases} = -A(2\theta(x - a/2) - 1)$$

where

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$$(6) \quad \theta(x) = \begin{cases} 1 & x < 0 \\ 0 & x > 0 \end{cases}$$

We've seen that the derivative of the step function can be taken as the delta function, so

$$(7) \quad \frac{d^2\Psi(x,0)}{dx^2} = -2A\delta(x - a/2)$$

Using the delta function directly, we get

$$(8) \quad \langle H \rangle = -\frac{\hbar^2}{2m} \int_0^a \Psi^* \frac{d^2\Psi}{dx^2} dx = \frac{\hbar^2 A^2 a}{m} \frac{1}{2} = \frac{6\hbar^2}{ma^2}$$

using $A^2 = 12/a^3$ from above. The final result is the same as that from summing the series as we did earlier.