INFINITE SQUARE WELL WITH TRIANGULAR INITIAL STATE USING DELTA FUNCTION

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 2.48.

Earlier, we looked at the case of a particle in the infinite square well with an initial wave function that was triangular:

$$\Psi(x,0)=\begin{cases}Ax & 0\le x\le\frac{a}{2}\\ A(a-x) & \frac{a}{2}\le x\le a\end{cases} \tag{1}$$

We find A from normalization:

$$\int_0^a|\Psi|^2dx=A^2a^3/12=1 \tag{2}$$

so

$$A=\frac{\sqrt{12}}{a^{3/2}} \tag{3}$$

Because the first derivative of $\Psi(x,0)$ is discontinuous at $x=0$, we might encounter problems in calculating the second derivative, which we need to find the mean value of the energy if we use integration, since this is

$$\langle E\rangle=\frac{\hbar^2}{2m}\int_0^a\Psi^*(x,0)\frac{d^2}{dx^2}\Psi(x,0)dx \tag{4}$$

We can express the first derivative of wave function as a step function $\theta(x)$:

$$\frac{d\Psi(x,0)}{dx}=\begin{cases}A & 0<x<a/2\\ -A & a/2<x<a\end{cases}=-A(2\theta(x-a/2)-1) \tag{5}$$

where

$$\theta(x)=\begin{cases}1 & x<0\\ 0 & x>0\end{cases} \tag{6}$$

We've seen that the derivative of the step function can be taken as the delta function, so

$$\frac{d^2\Psi(x,0)}{dx^2} = -2A\delta(x-a/2) \tag{7}$$

Using the delta function directly, we get

$$\langle H\rangle = -\frac{\hbar^2}{2m}\int_0^a \Psi^* \frac{d^2\Psi}{dx^2}dx = \frac{\hbar^2 A^2}{m}\frac{a}{2} = \frac{6\hbar^2}{ma^2} \tag{8}$$

using $A^2 = 12/a^3$ from above. The final result is the same as that from summing the series as we did earlier.