HILBERT SPACE - POWER FUNCTIONS

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 3.2.

In physics, Hilbert space is the vector space of all square-integrable functions. For the function $f(x) = x^{\nu}$ (with ν a real number) to be in Hilbert space on the interval $[0,1]$, it must be square-integrable over that interval. We have $\int_0^1 x^{2\nu} dx = x^{2\nu+1}/(2\nu+1)|_0^1$ which is finite provided $2\nu+1 > 0$ or $\nu > -1/2$.

The special case of $\nu = -1/2$ gives

$$\int_0^1 x^{-1} dx = \ln x|_0^1 \tag{1}$$

This diverges at $x = 0$, so $f(x) = x^{-1/2}$ is not in Hilbert space.

For $\nu = 1/2$, $f(x)$ and $xf(x)$ are both in Hilbert space, but $f'(x)$ is not, since its square integral gives the logarithm referred to above.