

## HILBERT SPACE - POWER FUNCTIONS

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Post date: 3 Sep 2012.

References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 3.2.

In physics, Hilbert space is the vector space of all square-integrable functions. For the function  $f(x) = x^\nu$  (with  $\nu$  a real number) to be in Hilbert space on the interval  $[0, 1]$ , it must be square-integrable over that interval. We have  $\int_0^1 x^{2\nu} dx = x^{2\nu+1}/(2\nu+1)|_0^1$  which is finite provided  $2\nu+1 > 0$  or  $\nu > -1/2$ .

The special case of  $\nu = -1/2$  gives

$$\int_0^1 x^{-1} dx = \ln x|_0^1 \quad (1)$$

This diverges at  $x = 0$ , so  $f(x) = x^{-1/2}$  is not in Hilbert space.

For  $\nu = 1/2$ ,  $f(x)$  and  $xf(x)$  are both in Hilbert space, but  $f'(x)$  is not, since its square integral gives the logarithm referred to above.