HERMITIAN OPERATORS: PERIODIC FUNCTION

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Section 3.2.2; Problem 3.8.

As a simple example of the fact that, for a hermitian operator, the eigenvalues are real and, for distinct eigenvalues, the eigenfunctions are orthogonal, we’ll consider again the periodic function we looked at earlier.

For the operator \( \frac{d^2}{d\phi^2} \), the eigenvalues are the negatives of the squares of the integers, so they are obviously real. The eigenfunctions are

\[
f = e^{in\phi}
\]

where \( n \) is any integer (positive, negative or zero).

To show they are orthogonal, we calculate

\[
\int_0^{2\pi} e^{in_1\phi} e^{-in_2\phi} d\phi = \int_0^{2\pi} e^{i(n_1-n_2)\phi} d\phi
\]

\[
= \frac{1}{i(n_1-n_2)} e^{i(n_1-n_2)\phi} \bigg|_0^{2\pi}
\]

\[
= 0
\]

provided \( n_1 \neq n_2 \).

Remember that the eigenvalues of this operator are actually \((in)^2 = -n^2\), so in this case the two eigenfunctions (one for \( in \) and the other for \(-in\)) for the same eigenvalue also happen to be orthogonal.

If we look at the operator \( id/d\phi \), we find the eigenvalues are all integers, with eigenfunctions \( e^{in\phi} \) since

\[
\frac{d}{d\phi} e^{in\phi} = -ne^{in\phi}
\]

This time, the eigenvalues are all non-degenerate. The eigenfunctions are all mutually orthogonal by the same argument as above.
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