HERMITIAN OPERATORS: PERIODIC FUNCTION

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Section 3.2.2; Problem 3.8.

As a simple example of the fact that, for a hermitian operator, the eigenvalues are real and, for distinct eigenvalues, the eigenfunctions are orthogonal, we'll consider again the periodic function we looked at earlier.

For the operator $d^2/d\phi^2$, the eigenvalues are the negatives of the squares of the integers, so they are obviously real. The eigenfunctions are

$$f = e^{in\phi} \tag{0.1}$$

where n is any integer (positive, negative or zero).

To show they are orthogonal, we calculate

$$\begin{aligned}
\int_0^{2\pi} e^{in_1\phi} e^{-in_2\phi} d\phi &= \int_0^{2\pi} e^{i(n_1-n_2)\phi} d\phi && (0.2)\\
&= \frac{1}{i(n_1-n_2)} e^{i(n_1-n_2)\phi}\Big|_0^{2\pi} && (0.3)\\
&= 0 && (0.4)
\end{aligned}$$

provided $n_1 \neq n_2$.

Remember that the eigenvalues of this operator are actually $(in)^2 = -n^2$, so in this case the two eigenfunctions (one for in and the other for $-in$) for the same eigenvalue also happen to be orthogonal.

If we look at the operator $id/d\phi$, we find the eigenvalues are all integers, with eigenfunctions $e^{in\phi}$ since

$$i\frac{d}{d\phi} e^{in\phi} = -ne^{in\phi} \tag{0.5}$$

This time, the eigenvalues are all non-degenerate. The eigenfunctions are all mutually orthogonal by the same argument as above.

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