INFINITE SQUARE WELL: MOMENTUM

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 3.10.

The stationary states of the infinite square well are

$$\psi_n(x)=\sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a} \tag{0.1}$$

where the well extends over the interval $x\in[0,a]$.

The momentum operator is

$$\hat{p}=-i\hbar\frac{d}{dx} \tag{0.2}$$

and we've seen that its eigenvalues are continuous in the case where we're considering an infinite interval. What happens if the interval is finite as in this case?

We can check the eigenvalue condition directly:

$$\begin{aligned}
\hat{p}\psi_n &= -i\hbar\frac{d}{dx}\sqrt{\frac{2}{a}}\sin\frac{n\pi x}{a} && (0.3)\\
&= -i\hbar\sqrt{\frac{2}{a}}\frac{n\pi}{a}\cos\frac{n\pi x}{a} && (0.4)\\
&= -i\frac{\hbar n\pi}{a}\cot\frac{n\pi x}{a}\psi_n(x) && (0.5)
\end{aligned}$$

Thus ψ_n is not an eigenfunction of momentum, since the momentum operator doesn't yield the original wave function multiplied by a constant.

The mean momentum is zero since it is

$$\begin{aligned}
\langle p\rangle &= \int_0^a \psi_n\hat{p}\psi_n dx && (0.6)\\
&= -i\frac{2n\pi\hbar}{a^2}\int_0^a \sin\frac{n\pi x}{a}\cos\frac{n\pi x}{a}dx && (0.7)\\
&= 0 && (0.8)
\end{aligned}$$

This means that the momentum is equally likely to be in either direction. The magnitude of the momentum is a constant, since this is a state with fixed energy, and $|p| = \sqrt{2mE} = n\pi\hbar/a$.