

## MOMENTUM SPACE: MEAN POSITION

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 3.12.

The momentum space wave function is the Fourier transform of the regular position space wave function:

$$\Phi(p, t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \Psi(x, t) e^{-ipx/\hbar} dx \quad (1)$$

In momentum space, the mean value of the momentum is simply

$$\langle p \rangle = \langle \Phi | p \Phi \rangle \quad (2)$$

$$= \int_{-\infty}^{\infty} \Phi^* p \Phi dp \quad (3)$$

The mean position turns out to be

$$\langle x \rangle = \int \Phi^* \left( -\frac{\hbar}{i} \frac{\partial}{\partial p} \right) \Phi dp \quad (4)$$

We can show this as follows. Starting with the definition above, we get

$$-\frac{\hbar}{i} \frac{\partial}{\partial p} \Phi = \frac{1}{\sqrt{2\pi\hbar}} \int x e^{-ipx/\hbar} \Psi dx \quad (5)$$

Substituting this together with the expression for  $\Phi^*$  from above into the RHS of 4 we get:

$$\int \Phi^* \left( -\frac{\hbar}{i} \frac{\partial}{\partial p} \right) \Phi dp = \frac{1}{2\pi\hbar} \int \left( \int e^{ipx'/\hbar} \Psi^*(x') dx' \right) \left( \int x e^{-ipx/\hbar} \Psi(x) dx \right) dp \quad (6)$$

$$= \frac{1}{2\pi\hbar} \int \left( \int \int e^{ip(x'-x)/\hbar} \Psi^*(x') x \Psi(x) dx' dx \right) dp \quad (7)$$

We can now do the integral over  $p$  first, and use the dodgy formula we obtained earlier, which expressed a delta function as an integral over a complex exponential:

$$\frac{1}{2\pi\hbar} \int e^{ip(x'-x)/\hbar} dp = \delta(x' - x) \quad (8)$$

Thus:

$$\int \Phi^* \left( -\frac{\hbar}{i} \frac{\partial}{\partial p} \right) \Phi dp = \int \int \delta(x' - x) \Psi^*(x') x \Psi(x) dx' dx \quad (9)$$

$$= \int \Psi^*(x) x \Psi(x) dx \quad (10)$$

$$= \langle x \rangle \quad (11)$$

QED.