

## MOMENTUM SPACE: MEAN POSITION

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 3.12.

The momentum space wave function is the Fourier transform of the regular position space wave function:

$$(0.1) \quad \Phi(p,t) = \frac{1}{\sqrt{2\pi\hbar}} \int_{-\infty}^{\infty} \Psi(x,t) e^{-ipx/\hbar} dx$$

In momentum space, the mean value of the momentum is simply

$$(0.2) \quad \langle p \rangle = \langle \Phi | p \Phi \rangle$$

$$(0.3) \quad = \int_{-\infty}^{\infty} \Phi^* p \Phi dp$$

The mean position turns out to be

$$(0.4) \quad \langle x \rangle = \int \Phi^* \left( -\frac{\hbar}{i} \frac{\partial}{\partial p} \right) \Phi dp$$

We can show this as follows. Starting with the definition above, we get

$$(0.5) \quad -\frac{\hbar}{i} \frac{\partial}{\partial p} \Phi = \frac{1}{\sqrt{2\pi\hbar}} \int x e^{-ipx/\hbar} \Psi dx$$

Substituting this together with the expression for  $\Phi^*$  from above into the RHS of 0.4 we get:

$$(0.6) \quad \int \Phi^* \left( -\frac{\hbar}{i} \frac{\partial}{\partial p} \right) \Phi dp = \frac{1}{2\pi\hbar} \int \left( \int e^{ipx'/\hbar} \Psi^*(x') dx' \right) \left( \int x e^{-ipx/\hbar} \Psi(x) dx \right) dp$$

$$(0.7) \quad = \frac{1}{2\pi\hbar} \int \left( \int \int e^{ip(x'-x)/\hbar} \Psi^*(x') x \Psi(x) dx' dx \right) dp$$

We can now do the integral over  $p$  first, and use the dodgy formula we obtained earlier, which expressed a delta function as an integral over a complex exponential:

$$(0.8) \quad \frac{1}{2\pi\hbar} \int e^{ip(x'-x)/\hbar} dp = \delta(x' - x)$$

Thus:

$$(0.9) \quad \int \Phi^* \left( -\frac{\hbar}{i} \frac{\partial}{\partial p} \right) \Phi dp = \int \int \delta(x' - x) \Psi^*(x') x \Psi(x) dx' dx$$

$$(0.10) \quad = \int \Psi^*(x) x \Psi(x) dx$$

$$(0.11) \quad = \langle x \rangle$$

QED.