In this post we'll have a look at an example involving the uncertainty principle. This makes use of the generic uncertainty principle for two observables $A$ and $B$:

$$\sigma_A^2 \sigma_B^2 \geq \left(\frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right)^2$$  \hspace{1cm} (1)

We can calculate an expression for a 'position-energy' uncertainty relation. For this we’ll need the commutator $[\hat{x}, \hat{H}]$, so we work that out first. Since $H$ involves a derivative, we use a test function $g$ on which to operate:

$$[\hat{x}, \hat{H}] g = -\frac{\hbar^2}{2m} x \frac{\partial^2 g}{\partial x^2} + x V g + \frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2} (x g) - x V g$$ \hspace{1cm} (2)

$$= \frac{\hbar^2}{2m} \left( -x \frac{\partial^2 g}{\partial x^2} + 2 \frac{\partial g}{\partial x} + x \frac{\partial^2 g}{\partial x^2} \right)$$ \hspace{1cm} (3)

$$= \frac{\hbar^2}{m} \frac{\partial g}{\partial x}$$ \hspace{1cm} (4)

$$= \frac{i\hbar}{m} (pg)$$ \hspace{1cm} (5)

From (1) with $\hat{A} = \hat{x}$ and $\hat{B} = \hat{H}$ we have

$$\sigma_x^2 \sigma_H^2 \geq \left(\frac{1}{2i} \langle [\hat{x}, \hat{H}] \rangle \right)^2$$ \hspace{1cm} (6)

$$= \frac{\hbar^2}{4m^2} \langle p \rangle^2$$ \hspace{1cm} (7)

so the uncertainty principle here becomes

$$\sigma_x \sigma_H \geq \frac{\hbar}{2m} |\langle p \rangle|$$ \hspace{1cm} (8)
For a stationary state, this doesn’t tell you much because the average position of the particle doesn’t change, so $\langle p \rangle = 0$. For linear combinations of stationary states, though, the average momentum will not be zero, so this condition is more than trivial.

PINGBACKS

Pingback: Uncertainty principle: rates of change of operators
Pingback: Energy-time uncertainty principle - example