

## UNCERTAINTY PRINCIPLE - EXAMPLES

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Sec 3.5; Problem 3.14.

In this post we'll have a look at an example involving the uncertainty principle. This makes use of the generic uncertainty principle for two observables  $A$  and  $B$ :

$$\sigma_A^2 \sigma_B^2 \geq \left( \frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right)^2 \quad (1)$$

We can calculate an expression for a 'position-energy' uncertainty relation. For this we'll need the commutator  $[\hat{x}, \hat{H}]$ , so we work that out first. Since  $H$  involves a derivative, we use a test function  $g$  on which to operate:

$$[\hat{x}, \hat{H}]g = -\frac{\hbar^2}{2m} x \frac{\partial^2 g}{\partial x^2} + xVg + -\frac{\hbar^2}{2m} \frac{\partial^2}{\partial x^2}(xg) - xVg \quad (2)$$

$$= \frac{\hbar^2}{2m} \left( -x \frac{\partial^2 g}{\partial x^2} + 2 \frac{\partial g}{\partial x} + x \frac{\partial^2 g}{\partial x^2} \right) \quad (3)$$

$$= \frac{\hbar^2}{m} \frac{\partial g}{\partial x} \quad (4)$$

$$= \frac{i\hbar}{m} (pg) \quad (5)$$

From 1 with  $\hat{A} = \hat{x}$  and  $\hat{B} = \hat{H}$  we have

$$\sigma_x^2 \sigma_H^2 \geq \left( \frac{1}{2i} \langle [\hat{x}, \hat{H}] \rangle \right)^2 \quad (6)$$

$$= \frac{\hbar^2}{4m^2} \langle p \rangle^2 \quad (7)$$

so the uncertainty principle here becomes

$$\sigma_x \sigma_H \geq \frac{\hbar}{2m} |\langle p \rangle| \quad (8)$$

For a stationary state, this doesn't tell you much because the average position of the particle doesn't change, so  $\langle p \rangle = 0$ . For linear combinations of stationary states, though, the average momentum will not be zero, so this condition is more than trivial.

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