

ENERGY-TIME UNCERTAINTY PRINCIPLE - EXAMPLE

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Section 3.6; Problem 3.20.

The generalized uncertainty principle for two operators is

$$(1) \quad \sigma_A^2 \sigma_B^2 \geq \left(\frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right)^2$$

In the derivation of the energy-time uncertainty principle we found that for an operator Q satisfies the equation

$$(2) \quad \frac{d}{dt} \langle Q \rangle = \frac{i}{\hbar} \langle [H, Q] \rangle + \left\langle \frac{\partial Q}{\partial t} \right\rangle$$

If we set $Q = x$ (the position operator) then we get (since this operator does not depend explicitly on time):

$$(3) \quad \frac{d}{dt} \langle x \rangle = \frac{i}{\hbar} \langle [H, x] \rangle$$

Using the generalized uncertainty principle from above, we have

$$(4) \quad \sigma_H^2 \sigma_x^2 \geq \left(-\frac{\hbar}{2} \frac{d \langle x \rangle}{dt} \right)^2$$

$$(5) \quad = \frac{\hbar^2}{4m^2} \left(m \frac{d \langle x \rangle}{dt} \right)^2$$

$$(6) \quad = \frac{\hbar^2}{4m^2} \langle p \rangle^2$$

This is the same relation as we discovered earlier by considering only the generalized uncertainty principle on its own.