# ENERGY-TIME UNCERTAINTY PRINCIPLE - EXAMPLE

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Post date: 27 Sep 2012.

References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Section 3.6; Problem 3.20.

The generalized uncertainty principle for two operators is

$$\sigma_A^2\sigma_B^2 \geq \left(\frac{1}{2i}\left\langle[\hat{A},\hat{B}]\right\rangle\right)^2 \tag{1}$$

In the derivation of the energy-time uncertainty principle we found that for an operator $Q$ satisfies the equation

$$\frac{d}{dt}\langle Q\rangle = \frac{i}{\hbar}\langle[H,Q]\rangle + \left\langle\frac{\partial Q}{\partial t}\right\rangle \tag{2}$$

If we set $Q = x$ (the position operator) then we get (since this operator does not depend explicitly on time):

$$\frac{d}{dt}\langle x\rangle = \frac{i}{\hbar}\langle[H,x]\rangle \tag{3}$$

Using the generalized uncertainty principle from above, we have

$$\begin{aligned}\sigma_H^2\sigma_x^2 &\geq \left(-\frac{\hbar}{2}\frac{d\langle x\rangle}{dt}\right)^2 &(4)\\ &= \frac{\hbar^2}{4m^2}\left(m\frac{d\langle x\rangle}{dt}\right)^2 &(5)\\ &= \frac{\hbar^2}{4m^2}\langle p\rangle^2 &(6)\end{aligned}$$

This is the same relation as we discovered earlier by considering only the generalized uncertainty principle on its own.