

ENERGY-TIME UNCERTAINTY PRINCIPLE - EXAMPLE

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Section 3.6; Problem 3.20.

The generalized uncertainty principle for two operators is

$$\sigma_A^2 \sigma_B^2 \geq \left(\frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right)^2 \quad (1)$$

In the derivation of the energy-time uncertainty principle we found that for an operator Q satisfies the equation

$$\frac{d}{dt} \langle Q \rangle = \frac{i}{\hbar} \langle [H, Q] \rangle + \left\langle \frac{\partial Q}{\partial t} \right\rangle \quad (2)$$

If we set $Q = x$ (the position operator) then we get (since this operator does not depend explicitly on time):

$$\frac{d}{dt} \langle x \rangle = \frac{i}{\hbar} \langle [H, x] \rangle \quad (3)$$

Using the generalized uncertainty principle from above, we have

$$\sigma_H^2 \sigma_x^2 \geq \left(-\frac{\hbar}{2} \frac{d\langle x \rangle}{dt} \right)^2 \quad (4)$$

$$= \frac{\hbar^2}{4m^2} \left(m \frac{d\langle x \rangle}{dt} \right)^2 \quad (5)$$

$$= \frac{\hbar^2}{4m^2} \langle p \rangle^2 \quad (6)$$

This is the same relation as we discovered earlier by considering only the generalized uncertainty principle on its own.