MATRIX ELEMENTS: EXAMPLE

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Section 3.6; Problem 3.22.

As a simple example of the bra-ket notation and the matrix representation of operators, suppose we have a three-dimensional vector space spanned by an orthonormal basis $|1\rangle,|2\rangle,|3\rangle$. We begin with a couple of vectors given by

$$|\alpha\rangle = i|1\rangle-2|2\rangle-i|3\rangle \tag{1}$$
$$|\beta\rangle = i|1\rangle+2|3\rangle \tag{2}$$

The corresponding bras are found by taking the complex conjugate:

$$\langle\alpha| = -i\langle 1|-2\langle 2|+i\langle 3| \tag{3}$$
$$\langle\beta| = -i\langle 1|+2\langle 3| \tag{4}$$

Remember that these bras are not vectors in their own right; rather, they are operators which acquire meaning only when they are applied to vectors to yield a complex number.

We can form the two inner products of these two vectors:

$$\langle\alpha|\beta\rangle = (-i)(i)+(-2)(0)+(i)(2) \tag{5}$$
$$= 1+2i \tag{6}$$
$$\langle\beta|\alpha\rangle = (-i)(i)+(0)(-2)+(2)(-i) \tag{7}$$
$$= 1-2i \tag{8}$$

Clearly $\langle\beta|\alpha\rangle=\langle\alpha|\beta\rangle^*$.

If we define an operator $\hat{A}=|\alpha\rangle\langle\beta|$ we can write it in terms of the basis above:

$$\hat{A}=|\alpha\rangle\langle\beta| \tag{9}$$
$$=(i|1\rangle-2|2\rangle-i|3\rangle)(-i\langle 1|+2\langle 3|) \tag{10}$$
$$=|1\rangle\langle 1|+2i|1\rangle\langle 3|+2i|2\rangle\langle 1|-4|2\rangle\langle 3|-|3\rangle\langle 1|-2i|3\rangle\langle 3| \tag{11}$$

From here, we can obtain its matrix elements in this basis by using the orthonomal property of the basis. For example

$$\langle 1|A|1\rangle = \langle 1|1\rangle\langle 1|1\rangle + 2i\langle 1|1\rangle\langle 3|1\rangle + 2i\langle 1|2\rangle\langle 1|1\rangle \tag{12}$$

$$-4\langle 1|2\rangle\langle 3|1\rangle - \langle 1|3\rangle\langle 1|1\rangle - 2i\langle 1|3\rangle\langle 3|1\rangle \tag{13}$$

$$= 1 \tag{14}$$

Doing similar calculations for the other elements, we get

$$\mathsf{A} = \begin{bmatrix} 1 & 0 & 2i \\ 2i & 0 & -4 \\ -1 & 0 & -2i \end{bmatrix} \tag{15}$$

The matrix is not hermitian, as it is not equal to its conjugate transpose.