

HAMILTONIAN IN TWO-LEVEL SYSTEM

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 3.23.

As another example of a hamiltonian in a two-level system, suppose we have a hamiltonian defined with respect to an orthonormal basis $|1\rangle, |2\rangle$ as:

$$(0.1) \quad H = E (|1\rangle\langle 1| - |2\rangle\langle 2| + |1\rangle\langle 2| + |2\rangle\langle 1|)$$

It is easiest to start with the matrix form of the Hamiltonian, which can be read from the definition of the operator using the orthonormal properties of the basis:

$$(0.2) \quad H = E \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$

The eigenvalues of the matrix are found from the usual determinant:

$$(0.3) \quad \begin{vmatrix} 1-\lambda & 1 \\ 1 & -1-\lambda \end{vmatrix} = 0$$

which has solutions

$$(0.4) \quad \lambda = \pm\sqrt{2}$$

The eigenvalues are therefore $\pm\sqrt{2}E$.

The eigenvectors are found from the equation (the factor of E cancels off both sides)

$$(0.5) \quad \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \pm\sqrt{2} \begin{pmatrix} a \\ b \end{pmatrix}$$

giving the condition $a + b = \pm\sqrt{2}a$ (the other condition is not independent). This can be written as $b = a(\pm\sqrt{2} - 1)$, so the two eigenvectors are, after normalization:

$$(0.6) \quad e_1 = \frac{1}{\sqrt{4-2\sqrt{2}}}(|1\rangle + (\sqrt{2}-1)|2\rangle)$$

$$(0.7) \quad e_2 = \frac{1}{\sqrt{4+2\sqrt{2}}}(|1\rangle + (-\sqrt{2}-1)|2\rangle)$$