

ANTI-HERMITIAN OPERATORS

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 3.26.

A hermitian operator is equal to its hermitian conjugate (which, remember, is the complex conjugate of the transpose of the matrix representing the operator). That is,

$$(0.1) \quad \hat{Q}^\dagger = \hat{Q}$$

This has the consequence that for inner products

$$(0.2) \quad \langle f | \hat{Q} g \rangle = \langle \hat{Q}^\dagger f | g \rangle$$

$$(0.3) \quad = \langle \hat{Q} f | g \rangle$$

An *anti-hermitian* operator is equal to the negative of its hermitian conjugate, that is

$$(0.4) \quad \hat{Q}^\dagger = -\hat{Q}$$

In inner products, this means

$$(0.5) \quad \langle f | \hat{Q} g \rangle = \langle \hat{Q}^\dagger f | g \rangle$$

$$(0.6) \quad = -\langle \hat{Q} f | g \rangle$$

The expectation value of an anti-hermitian operator is:

$$(0.7) \quad \langle f | \hat{Q} f \rangle = \langle \hat{Q}^\dagger f | f \rangle$$

$$(0.8) \quad = -\langle \hat{Q} f | f \rangle$$

$$(0.9) \quad = -\langle Q \rangle^*$$

But $\langle f | \hat{Q} f \rangle = \langle Q \rangle$ so $\langle Q \rangle = -\langle Q \rangle^*$, which means the expectation value must be pure imaginary.

For two hermitian operators \hat{Q} and \hat{R} we have

$$(0.10) \quad [\hat{Q}, \hat{R}] = \hat{Q}\hat{R} - \hat{R}\hat{Q}$$

$$(0.11) \quad [\hat{Q}, \hat{R}]^\dagger = \hat{R}^\dagger \hat{Q}^\dagger - \hat{Q}^\dagger \hat{R}^\dagger$$

$$(0.12) \quad = \hat{R}\hat{Q} - \hat{Q}\hat{R}$$

$$(0.13) \quad = [\hat{R}, \hat{Q}]$$

$$(0.14) \quad = -[\hat{Q}, \hat{R}]$$

where we have used the hermitian property $\hat{Q}^\dagger = \hat{Q}$ to get the third line. Thus the commutator of two hermitian operators is anti-hermitian.

If two operators \hat{S} and \hat{T} are anti-hermitian, a similar derivation shows that $[\hat{S}, \hat{T}]^\dagger = -[\hat{S}, \hat{T}]$ also:

$$(0.15) \quad [\hat{S}, \hat{T}] = \hat{S}\hat{T} - \hat{T}\hat{S}$$

$$(0.16) \quad [\hat{S}, \hat{T}]^\dagger = \hat{T}^\dagger \hat{S}^\dagger - \hat{S}^\dagger \hat{T}^\dagger$$

$$(0.17) \quad = (-\hat{T})(-\hat{S}) - (-\hat{S})(-\hat{T})$$

$$(0.18) \quad = -[\hat{S}, \hat{T}]$$

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