SEQUENTIAL MEASUREMENTS

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 3.27.

Suppose we have a two-state system, and there are two observables, A and B, that can be measured. Observable A has two eigenstates (eigenvectors, if you like) called ψ_1 and ψ_2 with eigenvalues (possible values that can be observed) of a_1 and a_2 respectively. For B, the eigenstates are ϕ_1 and ϕ_2, with eigenvalues b_1 and b_2.

Since both observables have a complete set of eigenstates, the state of the system can be expressed in terms of either set (in a way similar to that in which we can express states in either position or momentum space when we're considering one-dimensional states). In particular, we can express the eigenstates of one observable in terms of the eigenstates of the other. For example, we might have

$$\psi_1 = \frac{1}{5}(3\phi_1 + 4\phi_2) \tag{1}$$

$$\psi_2 = \frac{1}{5}(4\phi_1 - 3\phi_2) \tag{2}$$

Now suppose that observable A is measured and the value is found to be a_1. If a measurement gives a particular eigenvalue, then the state of the system collapses to the eigenfunction for that eigenvalue, so the state of the system immediately after the measurement is ψ_1.

If we now measure B, we can find the probability of getting either b_1 or b_2. Since $\psi_1 = \frac{1}{5}(3\phi_1 + 4\phi_2)$, the probability of getting b_1 is 9/25 and of b_2 is 16/25.

After measuring B, we measure A again. What is the probability of again getting a_1? The important point here is that we *did* measure B before attempting to measure A again. If we *hadn't* measured B, then the system would still be in state ψ_1, which means we would get a_1 with 100% certainty. However, measuring B will force the system into one of the eigenstates of B. If we don't know the outcome of the measurement of B, we must use conditional probabilities to calculate the probability of measuring a_1 after measuring B. This is

$$Prob(a_1) = Prob(a_1|b_1)Prob(b_1) + Prob(a_1|b_2)Prob(b_2) \tag{3}$$

where $Prob(a_1|b_1)$ means 'probability of a_1 given b_1'.

To get the conditional probabilities, we can invert the equations relating the two sets of eigenfunctions to get

$$\phi_1 = \frac{1}{5}(3\psi_1 + 4\psi_2) \tag{4}$$

$$\phi_2 = \frac{1}{5}(4\psi_1 - 3\psi_2) \tag{5}$$

From here we can see that

$$Prob(a_1|b_1) = \frac{9}{25} \tag{6}$$

$$Prob(a_1|b_2) = \frac{16}{25} \tag{7}$$

so, using the probabilites above for getting each measurement of B prior to the final measurement of A, the final probability of getting a_1 after measuring B is $(9/25)^2 + (16/25)^2 = \frac{337}{625} = 0.5392$.