

## SEQUENTIAL MEASUREMENTS

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 3.27.

Suppose we have a two-state system, and there are two observables,  $A$  and  $B$ , that can be measured. Observable  $A$  has two eigenstates (eigenvectors, if you like) called  $\psi_1$  and  $\psi_2$  with eigenvalues (possible values that can be observed) of  $a_1$  and  $a_2$  respectively. For  $B$ , the eigenstates are  $\phi_1$  and  $\phi_2$ , with eigenvalues  $b_1$  and  $b_2$ .

Since both observables have a complete set of eigenstates, the state of the system can be expressed in terms of either set (in a way similar to that in which we can express states in either position or momentum space when we're considering one-dimensional states). In particular, we can express the eigenstates of one observable in terms of the eigenstates of the other. For example, we might have

$$\psi_1 = \frac{1}{5}(3\phi_1 + 4\phi_2) \quad (1)$$

$$\psi_2 = \frac{1}{5}(4\phi_1 - 3\phi_2) \quad (2)$$

Now suppose that observable  $A$  is measured and the value is found to be  $a_1$ . If a measurement gives a particular eigenvalue, then the state of the system collapses to the eigenfunction for that eigenvalue, so the state of the system immediately after the measurement is  $\psi_1$ .

If we now measure  $B$ , we can find the probability of getting either  $b_1$  or  $b_2$ . Since  $\psi_1 = \frac{1}{5}(3\phi_1 + 4\phi_2)$ , the probability of getting  $b_1$  is  $9/25$  and of  $b_2$  is  $16/25$ .

After measuring  $B$ , we measure  $A$  again. What is the probability of again getting  $a_1$ ? The important point here is that we *did* measure  $B$  before attempting to measure  $A$  again. If we *hadn't* measured  $B$ , then the system would still be in state  $\psi_1$ , which means we would get  $a_1$  with 100% certainty. However, measuring  $B$  will force the system into one of the eigenstates of  $B$ . If we don't know the outcome of the measurement of  $B$ , we must use conditional probabilities to calculate the probability of measuring  $a_1$  after measuring  $B$ . This is

$$Prob(a_1) = Prob(a_1|b_1)Prob(b_1) + Prob(a_1|b_2)Prob(b_2) \quad (3)$$

where  $Prob(a_1|b_1)$  means 'probability of  $a_1$  given  $b_1$ '.

To get the conditional probabilities, we can invert the equations relating the two sets of eigenfunctions to get

$$\phi_1 = \frac{1}{5}(3\psi_1 + 4\psi_2) \quad (4)$$

$$\phi_2 = \frac{1}{5}(4\psi_1 - 3\psi_2) \quad (5)$$

From here we can see that

$$Prob(a_1|b_1) = \frac{9}{25} \quad (6)$$

$$Prob(a_1|b_2) = \frac{16}{25} \quad (7)$$

so, using the probabilities above for getting each measurement of  $B$  prior to the final measurement of  $A$ , the final probability of getting  $a_1$  after measuring  $B$  is  $(9/25)^2 + (16/25)^2 = \frac{337}{625} = 0.5392$ .