

SEQUENTIAL MEASUREMENTS

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 3.27.

Suppose we have a two-state system, and there are two observables, A and B , that can be measured. Observable A has two eigenstates (eigenvectors, if you like) called ψ_1 and ψ_2 with eigenvalues (possible values that can be observed) of a_1 and a_2 respectively. For B , the eigenstates are ϕ_1 and ϕ_2 , with eigenvalues b_1 and b_2 .

Since both observables have a complete set of eigenstates, the state of the system can be expressed in terms of either set (in a way similar to that in which we can express states in either position or momentum space when we're considering one-dimensional states). In particular, we can express the eigenstates of one observable in terms of the eigenstates of the other. For example, we might have

$$(1) \quad \psi_1 = \frac{1}{5}(3\phi_1 + 4\phi_2)$$

$$(2) \quad \psi_2 = \frac{1}{5}(4\phi_1 - 3\phi_2)$$

Now suppose that observable A is measured and the value is found to be a_1 . If a measurement gives a particular eigenvalue, then the state of the system collapses to the eigenfunction for that eigenvalue, so the state of the system immediately after the measurement is ψ_1 .

If we now measure B , we can find the probability of getting either b_1 or b_2 . Since $\psi_1 = \frac{1}{5}(3\phi_1 + 4\phi_2)$, the probability of getting b_1 is $9/25$ and of b_2 is $16/25$.

After measuring B , we measure A again. What is the probability of again getting a_1 ? The important point here is that we *did* measure B before attempting to measure A again. If we *hadn't* measured B , then the system would still be in state ψ_1 , which means we would get a_1 with 100% certainty. However, measuring B will force the system into one of the eigenstates of B . If we don't know the outcome of the measurement of B , we must use conditional probabilities to calculate the probability of measuring a_1 after measuring B . This is

$$(3) \quad \text{Prob}(a_1) = \text{Prob}(a_1|b_1)\text{Prob}(b_1) + \text{Prob}(a_1|b_2)\text{Prob}(b_2)$$

where $\text{Prob}(a_1|b_1)$ means 'probability of a_1 given b_1 '.

To get the conditional probabilities, we can invert the equations relating the two sets of eigenfunctions to get

$$(4) \quad \phi_1 = \frac{1}{5}(3\psi_1 + 4\psi_2)$$

$$(5) \quad \phi_2 = \frac{1}{5}(4\psi_1 - 3\psi_2)$$

From here we can see that

$$(6) \quad \text{Prob}(a_1|b_1) = \frac{9}{25}$$

$$(7) \quad \text{Prob}(a_1|b_2) = \frac{16}{25}$$

so, using the probabilities above for getting each measurement of B prior to the final measurement of A , the final probability of getting a_1 after measuring B is $(9/25)^2 + (16/25)^2 = \frac{337}{625} = 0.5392$.