

ENERGY-TIME UNCERTAINTY: AN ALTERNATIVE DEFINITION

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 3.32.

The energy-time uncertainty relation is derived by calculating the standard deviation of the energy σ_H , in terms of the rate of change of another observable Q , and the expression comes out to

$$(1) \quad \sigma_H \sigma_Q \geq \frac{\hbar}{2} \left| \frac{d}{dt} \langle Q \rangle \right|$$

From here we define $\Delta E \equiv \sigma_H$ and the uncertainty in time is

$$(2) \quad \Delta t \equiv \frac{\sigma_Q}{|d\langle Q \rangle / dt|}$$

This is roughly the amount of time it takes Q to change by one standard deviation.

A variation on the energy-time relation is to define $\Delta t \equiv \tau/\pi$, where τ is the time it takes for a wave function to change into another wave function that is orthogonal to the original function.

To see how this definition works in practice, we can start with a wave function that is a combination of two stationary states (for some arbitrary potential; the actual potential doesn't matter as we'll see). That is:

$$(3) \quad \Psi(x, 0) = \frac{1}{\sqrt{2}}(\psi_1(x) + \psi_2(x))$$

The wave function at the later time τ is then:

$$(4) \quad \Psi(x, \tau) = \frac{1}{\sqrt{2}}(\psi_1(x)e^{-iE_1\tau/\hbar} + \psi_2(x)e^{-iE_2\tau/\hbar})$$

For these two functions to be orthogonal, we must have:

$$(5) \quad \langle \Psi(x, \tau) | \Psi(x, 0) \rangle = 0$$

$$(6) \quad \frac{1}{2}(e^{iE_1\tau/\hbar} + e^{iE_2\tau/\hbar}) = 0$$

$$(7) \quad (1 + e^{i(E_2 - E_1)\tau/\hbar}) = 0$$

$$(8) \quad (E_2 - E_1)\tau/\hbar = \pi$$

$$(9) \quad \frac{\tau}{\pi}(E_2 - E_1) = \hbar$$

where we used the orthonormality of ψ_1 and ψ_2 in getting line 2.

Taking $\tau/\pi = \Delta t$ and $E_2 - E_1 = \Delta E$ gives the condition

$$(10) \quad \Delta E \Delta t = \hbar > \frac{\hbar}{2}$$

This is consistent with the original time-energy uncertainty principle. (Actually, to work out σ_H rather than just ΔE , we'd need to know the specific potential being used.)