

ENERGY-TIME UNCERTAINTY: AN ALTERNATIVE DEFINITION

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 3.32.

The energy-time uncertainty relation is derived by calculating the standard deviation of the energy σ_H , in terms of the rate of change of another observable Q , and the expression comes out to

$$\sigma_H \sigma_Q \geq \frac{\hbar}{2} \left| \frac{d}{dt} \langle Q \rangle \right| \quad (1)$$

From here we define $\Delta E \equiv \sigma_H$ and the uncertainty in time is

$$\Delta t \equiv \frac{\sigma_Q}{|d \langle Q \rangle / dt|} \quad (2)$$

This is roughly the amount of time it takes Q to change by one standard deviation.

A variation on the energy-time relation is to define $\Delta t \equiv \tau/\pi$, where τ is the time it takes for a wave function to change into another wave function that is orthogonal to the original function.

To see how this definition works in practice, we can start with a wave function that is a combination of two stationary states (for some arbitrary potential; the actual potential doesn't matter as we'll see). That is:

$$\Psi(x, 0) = \frac{1}{\sqrt{2}} (\psi_1(x) + \psi_2(x)) \quad (3)$$

The wave function at the later time τ is then:

$$\Psi(x, \tau) = \frac{1}{\sqrt{2}} (\psi_1(x) e^{-iE_1 \tau / \hbar} + \psi_2(x) e^{-iE_2 \tau / \hbar}) \quad (4)$$

For these two functions to be orthogonal, we must have:

$$\langle \Psi(x, \tau) | \Psi(x, 0) \rangle = 0 \quad (5)$$

$$\frac{1}{2}(e^{iE_1\tau/\hbar} + e^{iE_2\tau/\hbar}) = 0 \quad (6)$$

$$(1 + e^{i(E_2 - E_1)\tau/\hbar}) = 0 \quad (7)$$

$$(E_2 - E_1)\tau/\hbar = \pi \quad (8)$$

$$\frac{\tau}{\pi}(E_2 - E_1) = \hbar \quad (9)$$

where we used the orthonormality of ψ_1 and ψ_2 in getting line 2.

Taking $\tau/\pi = \Delta t$ and $E_2 - E_1 = \Delta E$ gives the condition

$$\Delta E \Delta t = \hbar > \frac{\hbar}{2} \quad (10)$$

This is consistent with the original time-energy uncertainty principle. (Actually, to work out σ_H rather than just ΔE , we'd need to know the specific potential being used.)