ENERGY-TIME UNCERTAINTY: AN ALTERNATIVE DEFINITION

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 3.32.

The energy-time uncertainty relation is derived by calculating the standard deviation of the energyσ_H, in terms of the rate of change of another observable Q, and the expression comes out to

$$\sigma_H\sigma_Q \geq \frac{\hbar}{2}\left|\frac{d}{dt}\langle Q\rangle\right| \tag{0.1}$$

From here we define $\Delta E \equiv \sigma_H$ and the uncertainty in time is

$$\Delta t \equiv \frac{\sigma_Q}{|d\langle Q\rangle/dt|} \tag{0.2}$$

This is roughly the amount of time it takes Q to change by one standard deviation.

A variation on the energy-time relation is to define $\Delta t \equiv \tau/\pi$, where τ is the time it takes for a wave function to change into another wave function that is orthogonal to the original function.

To see how this definition works in practice, we can start with a wave function that is a combination of two stationary states (for some arbitrary potential; the actual potential doesn't matter as we'll see). That is:

$$\Psi(x,0) = \frac{1}{\sqrt{2}}(\psi_1(x)+\psi_2(x)) \tag{0.3}$$

The wave function at the later time τ is then:

$$\Psi(x,\tau) = \frac{1}{\sqrt{2}}(\psi_1(x)e^{-iE_1\tau/\hbar}+\psi_2(x)e^{-iE_2\tau/\hbar}) \tag{0.4}$$

For these two functions to be orthogonal, we must have:

$$\langle\Psi(x,\tau)|\Psi(x,0)\rangle = 0 \tag{0.5}$$

$$\frac{1}{2}(e^{iE_1\tau/\hbar}+e^{iE_2\tau/\hbar}) = 0 \tag{0.6}$$

$$(1+e^{i(E_2-E_1)\tau/\hbar}) = 0 \tag{0.7}$$

$$(E_2-E_1)\tau/\hbar = \pi \tag{0.8}$$

$$\frac{\tau}{\pi}(E_2-E_1) = \hbar \tag{0.9}$$

where we used the orthonormality of ψ_1 and ψ_2 in getting line 2.

Taking $\tau/\pi = \Delta t$ and $E_2 - E_1 = \Delta E$ gives the condition

$$\Delta E\Delta t = \hbar > \frac{\hbar}{2} \tag{0.10}$$

This is consistent with the original time-energy uncertainty principle. (Actually, to work out σ_H rather than just ΔE, we'd need to know the specific potential being used.)