

EXTENDED UNCERTAINTY PRINCIPLE

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 3.36.

In deriving the uncertainty principle for two operators \hat{A} and \hat{B} , we found that

$$(0.1) \quad \sigma_A^2 \sigma_B^2 \geq |\langle f|g \rangle|^2$$

where

$$(0.2) \quad |f\rangle = |(\hat{A} - \langle A \rangle)\Psi\rangle$$

$$(0.3) \quad |g\rangle = |(\hat{B} - \langle B \rangle)\Psi\rangle$$

From there, we obtained the final inequality by considering only the imaginary part of $z = \langle f|g \rangle$, and got the uncertainty relation

$$(0.4) \quad \sigma_A^2 \sigma_B^2 \geq \left(\frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right)^2$$

If we retain the real part of z as well, we can get a stronger condition. We need to calculate $\Re(z) = (z + z^*)/2$, which we can do using results from the earlier post. We found there that

$$(0.5) \quad \langle f|g \rangle = \langle \hat{A}\hat{B} \rangle - \langle A \rangle \langle B \rangle$$

$$(0.6) \quad \langle g|f \rangle = \langle \hat{B}\hat{A} \rangle - \langle A \rangle \langle B \rangle$$

Therefore,

$$(0.7) \quad \frac{1}{2}(z + z^*) = \frac{1}{2}(\langle f|g \rangle + \langle g|f \rangle)$$

$$(0.8) \quad = \frac{1}{2}(\langle \hat{A}\hat{B} \rangle + \langle \hat{B}\hat{A} \rangle - 2\langle A \rangle \langle B \rangle)$$

and the extended uncertainty principle is

$$(0.9) \quad \sigma_A^2 \sigma_B^2 \geq \left(\frac{1}{2i} \langle [\hat{A}, \hat{B}] \rangle \right)^2 + \frac{1}{4} (\langle \hat{A}\hat{B} \rangle + \langle \hat{B}\hat{A} \rangle - 2\langle A \rangle \langle B \rangle)^2$$

In the special case where $\hat{B} = \hat{A}$, then $[\hat{A}, \hat{B}] = 0$ and the second term reduces to $2\langle A^2 \rangle - 2\langle A \rangle^2 = 2\sigma_A^2$, so the condition becomes $\sigma_A^4 \geq \sigma_A^4$, which doesn't tell you anything new.

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