EXTENDED UNCERTAINTY PRINCIPLE

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Post date: 15 Oct 2012.

References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 3.36.

In deriving the uncertainty principle for two operators $\hat{A}$ and $\hat{B}$, we found that

$$\sigma_A^2\sigma_B^2 \ge |\langle f|g\rangle|^2 \tag{1}$$

where

$$\begin{aligned} |f\rangle &= \left|(\hat{A}-\langle A\rangle)\Psi\right\rangle &(2)\\ |g\rangle &= \left|(\hat{B}-\langle B\rangle)\Psi\right\rangle &(3)\end{aligned}$$

From there, we obtained the final inequality by considering only the imaginary part of $z=\langle f|g\rangle$, and got the uncertainty relation

$$\sigma_A^2\sigma_B^2 \ge \left(\frac{1}{2i}\left\langle[\hat{A},\hat{B}]\right\rangle\right)^2 \tag{4}$$

If we retain the real part of z as well, we can get a stronger condition. We need to calculate $\Re(z)=(z+z^*)/2$, which we can do using results from the earlier post. We found there that

$$\begin{aligned} \langle f|g\rangle &= \left\langle\hat{A}\hat{B}\right\rangle-\langle A\rangle\langle B\rangle &(5)\\ \langle g|f\rangle &= \left\langle\hat{B}\hat{A}\right\rangle-\langle A\rangle\langle B\rangle &(6)\end{aligned}$$

Therefore,

$$\begin{aligned} \frac{1}{2}(z+z^*) &= \frac{1}{2}(\langle f|g\rangle+\langle g|f\rangle) &(7)\\ &= \frac{1}{2}(\langle\hat{A}\hat{B}\rangle+\langle\hat{B}\hat{A}\rangle-2\langle A\rangle\langle B\rangle) &(8)\end{aligned}$$

and the extended uncertainty principle is

$$\sigma_A^2\sigma_B^2 \ge \left(\frac{1}{2i}\left\langle[\hat{A},\hat{B}]\right\rangle\right)^2 + \frac{1}{4}\left(\langle\hat{A}\hat{B}\rangle + \langle\hat{B}\hat{A}\rangle - 2\langle A\rangle\langle B\rangle\right)^2 \tag{9}$$

In the special case where $\hat{B} = \hat{A}$, then $[\hat{A},\hat{B}] = 0$ and the second term reduces to $2\langle A^2\rangle - 2\langle A\rangle^2 = 2\sigma_A^2$, so the condition becomes $\sigma_A^4 \ge \sigma_A^4$, which doesn't tell you anything new.

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