## HAMILTONIAN FOR THREE-STATE SYSTEM

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 3.37.

Suppose we have a three-state hamiltonian whose matrix elements, relative to a certain basis, are

$$\mathsf{H}=\left[\begin{array}{ccc}a & 0 & b\\0 & c & 0\\b & 0 & a\end{array}\right] \tag{0.1}$$

To find the allowed energies and corresponding eigenstates, we find the eigenvalues and eigenvectors of $\mathsf{H}$.

$$\left|\begin{array}{ccc}a-E & 0 & b\\0 & c-E & 0\\b & 0 & a-E\end{array}\right|=0 \tag{0.2}$$

$$(a-E)^{2}(c-E)-b^{2}(c-E)=0 \tag{0.3}$$

This has 3 solutions:

$$\begin{aligned}E_{1} &= c && (0.4)\\E_{2} &= a+b && (0.5)\\E_{3} &= a-b && (0.6)\end{aligned}$$

The corresponding normalized eigenvectors are found in the usual way:

$$|1\rangle=\left(\begin{array}{c}0\\1\\0\end{array}\right) \tag{0.7}$$

$$|2\rangle=\frac{1}{\sqrt{2}}\left(\begin{array}{c}1\\0\\1\end{array}\right) \tag{0.8}$$

$$|3\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \tag{0.9}$$

If the system starts in the initial state which is the eigenvector $|1\rangle$, the time-dependent solution is

$$|\mathscr{S}(t)\rangle = e^{-iE_1 t/\hbar}|1\rangle \tag{0.10}$$
$$= e^{-ict/\hbar}|1\rangle \tag{0.11}$$

If the initial state is not one of the eigenvectors, such as

$$|\mathscr{S}(0)\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \tag{0.12}$$

we express this state as a linear combination of the last two eigenvectors:

$$|\mathscr{S}(0)\rangle = \frac{1}{\sqrt{2}}(|2\rangle - |3\rangle) \tag{0.13}$$

so the general solution is

$$|\mathscr{S}(t)\rangle = \frac{1}{\sqrt{2}}\left(e^{-iE_2 t/\hbar}|2\rangle - e^{-iE_3 t/\hbar}|3\rangle\right) \tag{0.14}$$
$$= \frac{1}{\sqrt{2}}\left(e^{-i(a+b)t/\hbar}|2\rangle - e^{-i(a-b)t/\hbar}|3\rangle\right) \tag{0.15}$$