

HAMILTONIAN FOR THREE-STATE SYSTEM

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Post date: 16 Oct 2012.

References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 3.37.

Suppose we have a three-state hamiltonian whose matrix elements, relative to a certain basis, are

$$H = \begin{bmatrix} a & 0 & b \\ 0 & c & 0 \\ b & 0 & a \end{bmatrix} \quad (1)$$

To find the allowed energies and corresponding eigenstates, we find the eigenvalues and eigenvectors of H.

$$\begin{vmatrix} a-E & 0 & b \\ 0 & c-E & 0 \\ b & 0 & a-E \end{vmatrix} = 0 \quad (2)$$

$$(a-E)^2(c-E) - b^2(c-E) = 0 \quad (3)$$

This has 3 solutions:

$$E_1 = c \quad (4)$$

$$E_2 = a+b \quad (5)$$

$$E_3 = a-b \quad (6)$$

The corresponding normalized eigenvectors are found in the usual way:

$$|1\rangle = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \quad (7)$$

$$|2\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} \quad (8)$$

$$|3\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix} \quad (9)$$

If the system starts in the initial state which is the eigenvector $|1\rangle$, the time-dependent solution is

$$|\mathcal{S}(t)\rangle = e^{-iE_1t/\hbar}|1\rangle \quad (10)$$

$$= e^{-ict/\hbar}|1\rangle \quad (11)$$

If the initial state is not one of the eigenvectors, such as

$$|\mathcal{S}(0)\rangle = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (12)$$

we express this state as a linear combination of the last two eigenvectors:

$$|\mathcal{S}(0)\rangle = \frac{1}{\sqrt{2}}(|2\rangle - |3\rangle) \quad (13)$$

so the general solution is

$$|\mathcal{S}(t)\rangle = \frac{1}{\sqrt{2}} \left(e^{-iE_2t/\hbar}|2\rangle - e^{-iE_3t/\hbar}|3\rangle \right) \quad (14)$$

$$= \frac{1}{\sqrt{2}} \left(e^{-i(a+b)t/\hbar}|2\rangle - e^{-i(a-b)t/\hbar}|3\rangle \right) \quad (15)$$

COMMENTS

Remark 1. Chris Kranenberg; Dec 6, 2017 4:46 PM

The solution can be taken a little further by replacing the vector notations with their matrices, perform the matrix addition to obtain a matrix solution with cosine and isine expressions.

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If you mean eqn (15), yes, you could factor out the $e^{-iat/\hbar}$ and then substitute from (8) and (9) to get the answer in terms of sin and cos of bt/\hbar .