HYDROGEN ATOM - WAVE FUNCTION EXAMPLES

A few more examples of working out the hydrogen atom wave functions. Using the formulas in the last example, we can get $R_{20}$. The recursion formula for $n=2, l=0$ is

$$c_{j+1} = \frac{2(j+1) - 4}{(j+1)(j+2)} c_j$$

(1)

The series has 2 terms, and we get $c_1 = -c_0$, so

$$R_{20}(r) = \frac{1}{r} u_{20}(r) = \frac{1}{r} \rho e^{-\rho} v_{20}(\rho) = \frac{1}{2a} e^{-\rho/2a} c_0 \left(1 - \frac{r}{2a}\right)$$

(2)

(3)

(4)

To find $c_0$ we normalize the radial function:

$$\int_0^\infty r^2 |R_{20}(r)|^2 dr = \int_0^\infty c_0^2 v_{20}^2 \left[ \frac{1}{2a} \left(1 - \frac{r}{2a}\right) \right]^2 e^{-\rho/2a} dr$$

(5)

$$= \frac{a}{2} c_0^2$$

(6)

$$= 1$$

(7)

So $c_0 = \sqrt{2/a}$ and $R_{20}(r)$ is

$$R_{20}(r) = \frac{1}{\sqrt{2a^{3/2}}} e^{-r/2a} \left(1 - \frac{r}{2a}\right)$$

(8)

The complete wave function is then

$$\psi_{200} = R_{20}(r) Y_{00}(\phi, \theta) = \frac{1}{\sqrt{8\pi}} a^{-3/2} \left(1 - \frac{r}{2a}\right) e^{-r/2a}$$

(9)

(10)
For $R_{21}(r)$, we have

$$c_{j+1} = \frac{2(j + 2) - 4}{(j + 1)(j + 4)} c_j$$  \hspace{1cm} (11)

This time, there is only a single term in the series, so we have

$$R_{21}(r) = \frac{1}{r} u_{21}(r)$$  \hspace{1cm} (12)

$$= \frac{1}{r} \rho^2 e^{-\rho} v_{21}(\rho)$$  \hspace{1cm} (13)

$$= \frac{r}{(2a)^2} e^{-r/2a} c_0$$  \hspace{1cm} (14)

Doing the normalization integral for $R_{21}(r)$ gives $c_0 = \sqrt{2/3a}$ which gives the final result

$$R_{21}(r) = \frac{r}{2\sqrt{6a^{5/2}}} e^{-r/2a}$$  \hspace{1cm} (15)

There are 3 wave functions corresponding to $n = 2$, $l = 1$, for which we need the spherical harmonics

$$Y_1^1 = -\left(\frac{3}{8\pi}\right)^{1/2} \sin \theta e^{i\phi}$$  \hspace{1cm} (16)

$$Y_{1}^{-1} = \left(\frac{3}{8\pi}\right)^{1/2} \sin \theta e^{-i\phi}$$  \hspace{1cm} (17)

$$Y_1^0 = \left(\frac{3}{4\pi}\right)^{1/2} \cos \theta$$  \hspace{1cm} (18)

The three wave functions are thus

$$\psi_{211} = -\frac{1}{8\sqrt{\pi}} \frac{r}{a^{5/2}} e^{-r/2a} \sin \theta e^{i\phi}$$  \hspace{1cm} (19)

$$\psi_{21-1} = \frac{1}{8\sqrt{\pi}} \frac{r}{a^{5/2}} e^{-r/2a} \sin \theta e^{-i\phi}$$  \hspace{1cm} (20)

$$\psi_{210} = \frac{1}{4\sqrt{2\pi}} \frac{r}{a^{5/2}} e^{-r/2a} \cos \theta$$  \hspace{1cm} (21)

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