HYDROGEN ATOM - WAVE FUNCTION EXAMPLES

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 4.11.

A few more examples of working out the hydrogen atom wave functions. Using the formulas in the last example, we can get R_{20}. The recursion formula for $n=2$, $l=0$ is

$$c_{j+1}=\frac{2(j+1)-4}{(j+1)(j+2)}c_j \tag{0.1}$$

The series has 2 terms, and we get $c_1=-c_0$, so

$$\begin{aligned}
R_{20}(r) &= \frac{1}{r}u_{20}(r) && (0.2)\\
&= \frac{1}{r}\rho e^{-\rho}v_{20}(\rho) && (0.3)\\
&= \frac{1}{2a}e^{-r/2a}c_0\left(1-\frac{r}{2a}\right) && (0.4)
\end{aligned}$$

To find c_0 we normalize the radial function:

$$\begin{aligned}
\int_0^\infty r^2|R_{20}(r)|^2 dr &= \int_0^\infty c_0^2 r^2\left[\frac{1}{2a}\left(1-\frac{1}{2a}\right)\right]^2 e^{-r/a}dr && (0.5)\\
&= \frac{a}{2}c_0^2 && (0.6)\\
&= 1 && (0.7)
\end{aligned}$$

So $c_0=\sqrt{2/a}$ and $R_{20}(r)$ is

$$R_{20}(r)=\frac{1}{\sqrt{2}a^{3/2}}e^{-r/2a}\left(1-\frac{r}{2a}\right) \tag{0.8}$$

The complete wave function is then

$$\begin{aligned}
\psi_{200} &= R_{20}(r)Y_{00}(\phi,\theta) && (0.9)\\
&= \frac{1}{\sqrt{8\pi}}a^{-3/2}\left(1-\frac{r}{2a}\right)e^{-r/2a} && (0.10)
\end{aligned}$$

For $R_{21}(r)$, we have

$$c_{j+1} = \frac{2(j+2)-4}{(j+1)(j+4)}c_j \tag{0.11}$$

This time, there is only a single term in the series, so we have

$$\begin{aligned} R_{21}(r) &= \frac{1}{r}u_{21}(r) && (0.12)\\ &= \frac{1}{r}\rho^2 e^{-\rho}v_{21}(\rho) && (0.13)\\ &= \frac{r}{(2a)^2}e^{-r/2a}c_0 && (0.14) \end{aligned}$$

Doing the normalization integral for $R_{21}(r)$ gives $c_0 = \sqrt{2/3a}$ which gives the final result

$$R_{21}(r) = \frac{r}{2\sqrt{6}a^{5/2}}e^{-r/2a} \tag{0.15}$$

There are 3 wave functions corresponding to $n = 2,\ l = 1$, for which we need the spherical harmonics

$$\begin{aligned} Y_1^1 &= -\left(\frac{3}{8\pi}\right)^{1/2}\sin\theta e^{i\phi} && (0.16)\\ Y_1^{-1} &= \left(\frac{3}{8\pi}\right)^{1/2}\sin\theta e^{-i\phi} && (0.17)\\ Y_1^0 &= \left(\frac{3}{4\pi}\right)^{1/2}\cos\theta && (0.18) \end{aligned}$$

The three wave functions are thus

$$\begin{aligned} \psi_{211} &= -\frac{1}{8\sqrt{\pi}}\frac{r}{a^{5/2}}e^{-r/2a}\sin\theta e^{i\phi} && (0.19)\\ \psi_{21-1} &= \frac{1}{8\sqrt{\pi}}\frac{r}{a^{5/2}}e^{-r/2a}\sin\theta e^{-i\phi} && (0.20)\\ \psi_{210} &= \frac{1}{4\sqrt{2\pi}}\frac{r}{a^{5/2}}e^{-r/2a}\cos\theta && (0.21) \end{aligned}$$

PINGBACKS

Pingback: Hydrogen atom - mean radius of electron position
Pingback: Hydrogen atom - mixed initial state and mean potential energy
Pingback: Probability current in 3-d