HYDROGEN ATOM - MOST PROBABLE DISTANCE OF ELECTRON

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 4.14.

We worked out the mean distance of the electron from the nucleus in the hydrogen atom, but is this also the most probable distance?

The probability that the electron is found between r and $r+dr$ in the ground state is $|R_{10}|^2 r^2 dr = (4/a^3)e^{-2r/a}r^2 dr$. The most likely position is thus the maximum of this function which can be found by differentiation.

$$\frac{d}{dr}(r^2 e^{-2r/a}) = 2re^{-2r/a}(1-r/a) \tag{0.1}$$

$$= 0 \tag{0.2}$$

$$r_{crit} = 0,\ a \tag{0.3}$$

The value of 0 is a minimum point, so the required value is

$$r_{max} = a \tag{0.4}$$

For higher energy levels, we would expect the most probable distance to increase, and this is in fact true. Using the radial function for the $n=2$, $l=1$ state, we have

$$R_{21}(r) = \frac{1}{\sqrt{24}}\frac{r}{a^{5/2}}e^{-r/2a} \tag{0.5}$$

$$\frac{d}{dr}\left(r^2|R_{21}|^2\right) = \frac{1}{24a^5}e^{-r/a}\left(4-\frac{r}{a}\right)r^3 \tag{0.6}$$

Setting this to 0 gives a triple root at $r=0$ (minimum) and a single root at $r=4a$ (the maximum probability).