

EARTH-SUN SYSTEM AS A QUANTUM ATOM

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 4.17.

An amusing little problem is to consider the Earth-Sun system as a giant analogue to the quantum model of the hydrogen atom. Since the gravitational potential is inverse- r , the same as the electromagnetic force, the calculations translate quite easily. The potential is

$$(0.1) \quad V = -G \frac{mM}{r}$$

where G is the gravitational constant, $G = 6.673 \times 10^{-11} m^3 kg^{-1} s^{-2}$.

The Bohr radius for the Earth can be found by replacing e^2 by mM , and $1/4\pi\epsilon_0$ by G .

$$(0.2) \quad a_g = \frac{\hbar^2}{Gm^2M}$$

In MKS, the values are $\hbar = 1.0546 \times 10^{-34} m^2 kg/s$, mass of the Earth is $m = 5.9742 \times 10^{24}$ kg, and mass of the Sun is $M = 1.98892 \times 10^{30}$ kg. Plugging the numbers gives

$$(0.3) \quad a_g = 2.349 \times 10^{-138} m$$

(that is, small).

From Bohr's formula for the energy levels, doing the same replacements as in above, we get

$$(0.4) \quad E_n = - \left[\frac{m}{2\hbar^2} (GmM)^2 \right] \frac{1}{n^2}$$

In classical physics, the energy of a planet in a circular orbit of radius r_0 is the sum of the gravitational potential energy and the kinetic energy. The gravitational potential is

$$(0.5) \quad V_g = -\frac{GmM}{r_0}$$

The kinetic energy can be obtained from equating the planet's centripetal force mv^2/r_0 to the gravitational force GmM/r_0^2 .

$$(0.6) \quad \frac{mv^2}{r_0} = \frac{GmM}{r_0^2}$$

$$(0.7) \quad \frac{1}{2}mv^2 = \frac{GmM}{2r_0}$$

$$(0.8) \quad E = V_g + \frac{1}{2}mv^2$$

$$(0.9) \quad = -\frac{GmM}{2r_0}$$

Equating this classical version of the energy with the Bohr energy from 0.4 we get

$$(0.10) \quad n^2 = \frac{Gm^2M}{\hbar^2}r_0$$

$$(0.11) \quad = \frac{r_0}{a_g}$$

using 0.2. For the Earth, $r_0 = 1.49598 \times 10^{11}$ m, so the Earth's quantum number is around 2.524×10^{74} .

The difference between energy levels for the Earth is very small, so we can use an approximation:

$$(0.12) \quad \frac{1}{(n-1)^2} - \frac{1}{n^2} = \frac{n^2 - (n-1)^2}{n^2(n-1)^2}$$

$$(0.13) \quad = \frac{2n-1}{n^2(n-1)^2}$$

$$(0.14) \quad = \frac{2 - \frac{1}{n}}{n(n-1)^2}$$

$$(0.15) \quad \approx \frac{2}{n^3}$$

for large n . Therefore, the energy released by a transition to the next lower level is

$$(0.16) \quad \Delta E = \frac{m(GmM)^2}{2\hbar^2} \frac{2}{n^3}$$

$$(0.17) \quad = 2.1 \times 10^{-41} J$$

The frequency of the photon is thus $\nu = \Delta E/h = 3.168 \times 10^{-8} \text{sec}^{-1}$, and the wavelength is

$$(0.18) \quad \lambda = \frac{c}{\nu}$$

$$(0.19) \quad = 9.469 \times 10^{15} m$$

$$(0.20) \quad \approx 1 l.y.$$

This value results from the fact that the Earth's frequency of oscillation is once per year, as that is the time it takes for the Earth to complete a single orbit. The photon or graviton takes a full year to be emitted, so its wavelength will be 1 light year.