

EARTH-SUN SYSTEM AS A QUANTUM ATOM

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 4.17.

An amusing little problem is to consider the Earth-Sun system as a giant analogue to the quantum model of the hydrogen atom. Since the gravitational potential is inverse- r , the same as the electromagnetic force, the calculations translate quite easily. The potential is

$$V = -G \frac{mM}{r} \quad (1)$$

where G is the gravitational constant, $G = 6.673 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2}$.

The Bohr radius for the Earth can be found by replacing e^2 by mM , and $1/4\pi\epsilon_0$ by G .

$$a_g = \frac{\hbar^2}{Gm^2M} \quad (2)$$

In MKS, the values are $\hbar = 1.0546 \times 10^{-34} \text{ m}^2 \text{ kg} / \text{s}$, mass of the Earth is $m = 5.9742 \times 10^{24} \text{ kg}$, and mass of the Sun is $M = 1.98892 \times 10^{30} \text{ kg}$. Plugging the numbers gives

$$a_g = 2.349 \times 10^{-138} \text{ m} \quad (3)$$

(that is, small).

From Bohr's formula for the energy levels, doing the same replacements as in above, we get

$$E_n = - \left[\frac{m}{2\hbar^2} (GmM)^2 \right] \frac{1}{n^2} \quad (4)$$

In classical physics, the energy of a planet in a circular orbit of radius r_0 is the sum of the gravitational potential energy and the kinetic energy. The gravitational potential is

$$V_g = - \frac{GmM}{r_0} \quad (5)$$

The kinetic energy can be obtained from equating the planet's centripetal force mv^2/r_0 to the gravitational force GmM/r_0^2 .

$$\frac{mv^2}{r_0} = \frac{GmM}{r_0^2} \quad (6)$$

$$\frac{1}{2}mv^2 = \frac{GmM}{2r_0} \quad (7)$$

$$E = V_g + \frac{1}{2}mv^2 \quad (8)$$

$$= -\frac{GmM}{2r_0} \quad (9)$$

Equating this classical version of the energy with the Bohr energy from 4 we get

$$n^2 = \frac{Gm^2M}{\hbar^2}r_0 \quad (10)$$

$$= \frac{r_0}{a_g} \quad (11)$$

using 2. For the Earth, $r_0 = 1.49598 \times 10^{11}$ m, so the Earth's quantum number is around 2.524×10^{74} .

The difference between energy levels for the Earth is very small, so we can use an approximation:

$$\frac{1}{(n-1)^2} - \frac{1}{n^2} = \frac{n^2 - (n-1)^2}{n^2(n-1)^2} \quad (12)$$

$$= \frac{2n-1}{n^2(n-1)^2} \quad (13)$$

$$= \frac{2 - \frac{1}{n}}{n(n-1)^2} \quad (14)$$

$$\approx \frac{2}{n^3} \quad (15)$$

for large n . Therefore, the energy released by a transition to the next lower level is

$$\Delta E = \frac{m(GmM)^2}{2\hbar^2} \frac{2}{n^3} \quad (16)$$

$$= 2.1 \times 10^{-41} J \quad (17)$$

The frequency of the photon is thus $\nu = \Delta E/h = 3.168 \times 10^{-8} \text{sec}^{-1}$, and the wavelength is

$$\lambda = \frac{c}{\nu} \quad (18)$$

$$= 9.469 \times 10^{15} m \quad (19)$$

$$\approx 1 l.y. \quad (20)$$

This value results from the fact that the Earth's frequency of oscillation is once per year, as that is the time it takes for the Earth to complete a single orbit. The photon or graviton takes a full year to be emitted, so its wavelength will be 1 light year.