

SPHERICAL HARMONIC USING THE RAISING OPERATOR

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education Problem 4.23.

Earlier, we found Y_2^1 using associated Legendre functions:

$$(0.1) \quad Y_2^1(\theta, \phi) = \sqrt{\frac{15}{8\pi}} e^{i\phi} \sin \theta \cos \theta$$

We can use the raising operator to find Y_2^2 from this. In spherical coordinates, the raising operator is

$$(0.2) \quad L_+ = \hbar e^{i\phi} \left[\frac{\partial}{\partial \theta} + i \cot \theta \frac{\partial}{\partial \phi} \right]$$

Applying the raising operator to Y_2^1 gives

$$(0.3) \quad L_+ Y_2^1 = -\sqrt{\frac{15}{8\pi}} \hbar e^{i\phi} ((\cos^2 \theta - \sin^2 \theta) e^{i\phi} - \cot \theta \sin \theta \cos \theta e^{i\phi})$$

$$(0.4) \quad = \sqrt{\frac{15}{8\pi}} \hbar e^{2i\phi} \sin^2 \theta$$

$$(0.5) \quad = A_2^1 Y_2^2$$

where A_2^1 is the constant derived earlier for the raising operator:

$$(0.6) \quad A_l^m = \hbar \sqrt{(l-m)(l+m+1)}$$

In this case

$$(0.7) \quad A_2^1 = 2\hbar$$

Therefore

$$(0.8) \quad Y_2^2 = \sqrt{\frac{15}{32\pi}} e^{2i\phi} \sin^2 \theta$$

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