SPHERICAL HARMONIC USING THE RAISING OPERATOR

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education Problem 4.23.

Earlier, we found Y_2^1 using associated Legendre functions:

$$Y_2^1(\theta,\phi)=\sqrt{\frac{15}{8\pi}}e^{i\phi}\sin\theta\cos\theta \tag{1}$$

We can use the raising operator to find Y_2^2 from this. In spherical coordinates, the raising operator is

$$L_+=\hbar e^{i\phi}\left[\frac{\partial}{\partial\theta}+i\cot\theta\frac{\partial}{\partial\phi}\right] \tag{2}$$

Applying the raising operator to Y_2^1 gives

$$\begin{aligned}
L_+Y_2^1 &= -\sqrt{\frac{15}{8\pi}}\hbar e^{i\phi}((\cos^2\theta-\sin^2\theta)e^{i\phi}-\cot\theta\sin\theta\cos\theta e^{i\phi}) && (3)\\
&= \sqrt{\frac{15}{8\pi}}\hbar e^{2i\phi}\sin^2\theta && (4)\\
&= A_2^1Y_2^2 && (5)
\end{aligned}$$

where A_2^1 is the constant derived earlier for the raising operator:

$$A_l^m=\hbar\sqrt{(l-m)(l+m+1)} \tag{6}$$

In this case

$$A_2^1=2\hbar \tag{7}$$

Therefore

$$Y_2^2=\sqrt{\frac{15}{32\pi}}e^{2i\phi}\sin^2\theta \tag{8}$$

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