SPHERICAL HARMONIC USING THE RAISING OPERATOR

Earlier, we found

\[ Y_{12}^1(\theta, \phi) = \sqrt{\frac{15}{8\pi}} e^{i\phi} \sin \theta \cos \theta \]  

(1)

We can use the raising operator to find \( Y_{22}^2 \) from this. In spherical coordinates, the raising operator is

\[ L_+ = \hbar e^{i\phi} \left[ \frac{\partial}{\partial \theta} + i \cot \theta \frac{\partial}{\partial \phi} \right] \]  

(2)

Applying the raising operator to \( Y_{12}^1 \) gives

\[ L_+ Y_{12}^1 = -\sqrt{\frac{15}{8\pi}} \hbar e^{i\phi} ((\cos^2 \theta - \sin^2 \theta) e^{i\phi} - \cot \theta \sin \theta \cos \theta e^{i\phi}) \]  

(3)

\[ = \sqrt{\frac{15}{8\pi}} \hbar e^{2i\phi} \sin^2 \theta \]  

(4)

\[ = A_{22}^1 Y_{22}^2 \]  

(5)

where \( A_{22}^1 \) is the constant derived earlier for the raising operator:

\[ A_{lm}^m = \hbar \sqrt{(l-m)(l+m+1)} \]  

(6)

In this case

\[ A_{22}^1 = 2\hbar \]  

(7)

Therefore

\[ Y_{22}^2 = \sqrt{\frac{15}{32\pi}} e^{2i\phi} \sin^2 \theta \]  

(8)

PINGBACKS

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