RIGID ROTOR IN QUANTUM MECHANICS

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References: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education Problem 4.24.

A rigid rotor is a system in which we have two particles, each of mass m, at the ends of a rigid rod of length a. The centre of the rod is fixed, but the rod is free to rotate about its centre in any direction (so it's not rotating on a fixed axis). Assuming no potential energy, the energy of the system is, for the two masses

$$E = 2\frac{p^2}{2m} = \frac{p^2}{m} \tag{1}$$

Since the particles are constrained to have rotational motion only, and the rod is fixed, $\mathbf{r}$ is always perpendicular to $\mathbf{p}$, so taking $\mathbf{p}$ to be the momentum of one of the masses, $|\mathbf{L}| = 2|\mathbf{r}||\mathbf{p}| = 2(a/2)p = ap$ and $L^2 = a^2p^2$ so

$$E = \frac{L^2}{ma^2} \tag{2}$$

We know the eigenvalues of L^2 are $\hbar^2 n(n+1)$ for $n = 0, 1, 2, \ldots$ so

$$E_n = \frac{\hbar^2 n(n+1)}{ma^2} \tag{3}$$

Since E is directly proportional to L^2, the eigenfunctions are just the spherical harmonics, and state n therefore has degeneracy $2n+1$.