ELECTRON AS A CLASSICAL SPINNING SPHERE

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Sec 4.4 & Problem 4.25.

Angular momentum can be written as $L = I\omega$, where I is the moment of inertia and ω is the angular velocity. For a solid sphere of mass m and radius r, $I = 2mr^2/5$. The equatorial speed on the sphere is $v = r\omega$, so

$$L = \frac{\hbar}{2} \tag{0.1}$$

$$= \frac{2}{5}mr_c^2\omega \tag{0.2}$$

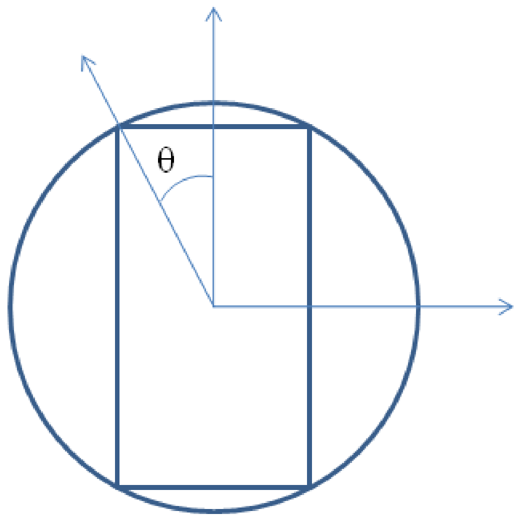
$$v = r\omega \tag{0.3}$$

$$= \frac{5}{4}\frac{\hbar}{mr} \tag{0.4}$$

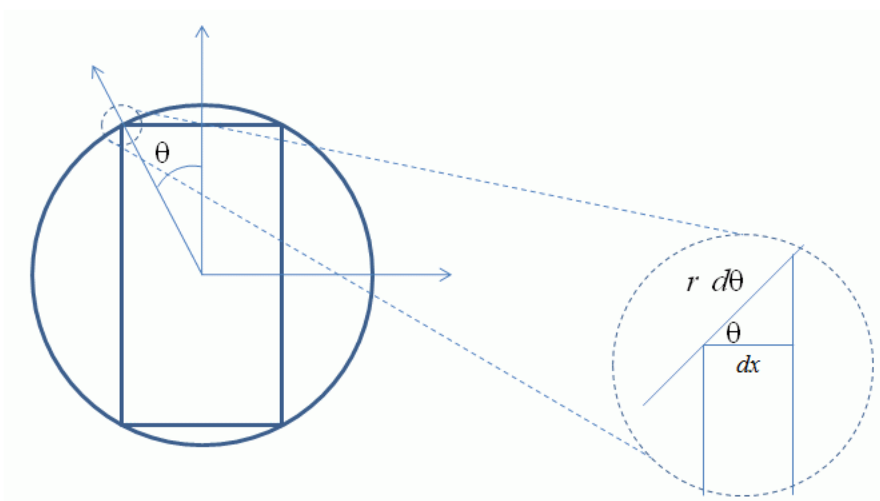
$$= \frac{5}{4}\frac{4\pi\varepsilon_0 c^2\hbar}{e^2} \tag{0.5}$$

In SI units, $\hbar = 1.0546\times10^{-34} m^2kg/s$, $1/4\pi\varepsilon_0 = 8.9876\times10^9\, N\, m^2/C^2$, $c = 3\times10^8 m/s$, $e = 1.6\times10^{-19}C$. Plugging in the numbers gives a speed of 5.16×10^{10} m s^{-1}, which is of course larger than the speed of light, so the classical analysis doesn't make sense.

As an extra, here is a derivation of the moment of inertia of a sphere using cylindrical shells, which isn't the usual derivation. Suppose we divide the sphere into a number of concentric cylindrical shells, whose axes coincide with the axis of rotation of the sphere (see diagram for side view).



All points in the cylindrical shell are the same distance $r\sin\theta$ from the axis of the sphere. Since the radius of the cylinder is $r\sin\theta$ and its height is $2r\cos\theta$ the volume of the shell is $(2\pi r\sin\theta)(2r\cos\theta)dx$ where dx is the thickness of the shell. To get the thickness of the shell, consider the following diagram.



On the right is a magnified view of the top of the shell. If we consider an increment in the angle θ, this generates an infinitesimal distance $r\cdot d\theta$ along the sphere. We want the horizontal increment corresponding to this distance. The angle between the horizontal and the tangent to the sphere at this point is θ (since these two lines are perpendicular to the two lines that generated θ in the diagram on the left), so the horizontal increment is $dx = r\cos\theta d\theta$. The volume of the shell is therefore

$$dV = 4\pi r^3 \sin\theta \cos^2\theta d\theta \tag{0.6}$$

The mass of the shell is the density ρ multiplied by the volume, or ρdV. The moment of inertia of the shell is

$$\begin{aligned} dI &= dV(r\sin\theta)^2 && (0.7)\\ &= 4\pi r^5 \rho \sin^3\theta \cos^2\theta d\theta && (0.8)\\ &= 4\pi r^5 \rho \sin\theta(\cos^2\theta - \cos^4\theta) d\theta && (0.9) \end{aligned}$$

In this last form, the integral is easy, so we get

$$\begin{aligned} I &= 4\pi r^5 \rho \int_0^{\pi/2} \sin\theta(\cos^2\theta - \cos^4\theta) d\theta && (0.10)\\ &= \frac{8}{15}\pi r^5 \rho && (0.11) \end{aligned}$$

The mass of the sphere is $m = 4\pi r^3 \rho/3$, so this formula becomes

$$I = \frac{2}{5} m r^2 \tag{0.12}$$