

SPIN - EXPECTATION VALUES OF COMPONENTS

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 4.28.

The most general spin state for a spin 1/2 particle is

$$\chi = \begin{bmatrix} a \\ b \end{bmatrix} \quad (1)$$

From this we can calculate the expectation values of the spin components. By direct matrix multiplication using the spin matrices, we get

$$\langle S_x \rangle = \frac{\hbar}{2} (a^* \ b^*) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \frac{\hbar}{2} (a^*b + b^*a) \quad (2)$$

This is a real quantity since it is the sum of a number and its complex conjugate. Similarly

$$\langle S_y \rangle = \frac{\hbar}{2} (a^* \ b^*) \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = i \frac{\hbar}{2} (b^*a - a^*b) \quad (3)$$

This is also real, since the difference of a number and its complex conjugate is pure imaginary, so multiplying by i gives a real number.

Finally,

$$\langle S_z \rangle = \frac{\hbar}{2} (a^* \ b^*) \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \frac{\hbar}{2} (|a|^2 - |b|^2) \quad (4)$$

Since $S_x^2 = S_y^2 = S_z^2 = \frac{\hbar^2}{4} \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$, $\langle S_x^2 \rangle = \langle S_y^2 \rangle = \langle S_z^2 \rangle = |\chi|^2 \hbar^2/4 = \hbar^2/4$, and $\langle S_x^2 \rangle + \langle S_y^2 \rangle + \langle S_z^2 \rangle = 3\hbar^2/4 = \langle S^2 \rangle$ which agrees with our original result.

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