

SPIN 1

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 4.31.

We can apply the methods for analyzing spin 1/2 to higher spin states as well. Here we'll have a look at spin 1. For spin 1, we know there will be 3 eigenstates of S_z ($m = 1, 0, -1$). Following the derivation for spin 1/2 we have

$$\chi = a\chi_1 + b\chi_0 + c\chi_{-1} = a \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + b \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} + c \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} \quad (1)$$

First, we know that $S^2\chi_j = \hbar^2 s(s+1)\chi_j = 2\hbar^2\chi_j$ for $j = 1, 0, -1$, so

$$S^2 = 2\hbar^2 \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \quad (2)$$

Since $S_z\chi_j = j\hbar\chi_j$ for $j = 1, 0, -1$

$$S_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix} \quad (3)$$

To get S_x and S_y we first get the raising and lowering matrices S_+ and S_- . The actions of these matrices on the eigenspinors is

$$S_{\pm}|s m_s\rangle = \hbar\sqrt{s(s+1) - m(m \pm 1)}|s m_s \pm 1\rangle \quad (4)$$

So we get $S_+\chi_1 = 0$, $S_+\chi_0 = \sqrt{2}\hbar\chi_1$ and $S_+\chi_{-1} = \sqrt{2}\hbar\chi_0$, from which we can write

$$S_+ = \sqrt{2}\hbar \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \quad (5)$$

By a similar argument we get

$$S_- = \sqrt{2}\hbar \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad (6)$$

We can now get S_x and S_y

$$S_x = \frac{1}{2}(S_+ + S_-) = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \quad (7)$$

$$S_y = \frac{1}{2i}(S_+ - S_-) = \frac{\hbar}{\sqrt{2}} \begin{pmatrix} 0 & -i & 0 \\ i & 0 & -i \\ 0 & i & 0 \end{pmatrix} \quad (8)$$

As a check, note that the eigenvalues of all of S_z , S_x and S_y are \hbar , 0 and $-\hbar$, as can be checked by direct calculation. For example, for S_x :

$$\begin{vmatrix} -\lambda & \frac{\hbar}{\sqrt{2}} & 0 \\ \frac{\hbar}{\sqrt{2}} & -\lambda & \frac{\hbar}{\sqrt{2}} \\ 0 & \frac{\hbar}{\sqrt{2}} & -\lambda \end{vmatrix} = -\lambda \left(\lambda^2 - \frac{\hbar^2}{2} \right) - \frac{\hbar}{\sqrt{2}} \left(-\lambda \frac{\hbar}{\sqrt{2}} \right) = 0 \quad (9)$$

This gives the cubic equation

$$-\lambda^3 + \lambda\hbar^2 = 0 \quad (10)$$

which has the roots $\lambda = 0, \pm\hbar$.

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