

## HARMONIC OSCILLATOR IN 3-D - RECTANGULAR COORDINATES

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Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 4.38.

The 3-d harmonic oscillator can be solved in rectangular coordinates by separation of variables. The Schrödinger equation to be solved for the 3-d harmonic oscillator is

$$-\frac{\hbar^2}{2m}\nabla^2\psi + \frac{1}{2}m\omega^2(x^2 + y^2 + z^2)\psi = E\psi \quad (1)$$

To use separation of variables we define

$$\psi(x, y, z) = \xi(x)\eta(y)\zeta(z) \quad (2)$$

Dividing 1 through by this product we get

$$-\frac{\hbar^2}{2m}\frac{\xi''}{\xi} + \frac{1}{2}m\omega^2x^2 - \frac{\hbar^2}{2m}\frac{\eta''}{\eta} + \frac{1}{2}m\omega^2y^2 - \frac{\hbar^2}{2m}\frac{\zeta''}{\zeta} + \frac{1}{2}m\omega^2z^2 = E \quad (3)$$

where the double prime notation indicates the second derivative of a function with respect to its independent variable, so  $\xi'' = d^2\xi/dx^2$ , etc.

We now have three groups of two terms each of which depends on only one of the variables  $x$ ,  $y$  and  $z$ , and the sum of all these terms is the constant  $E$ . We can therefore use the usual argument that each group of two terms must be a constant on its own, so the 3-d equation reduces to the sum of three 1-d harmonic oscillators. From the analysis of the 1-d harmonic oscillator, we know that each of these will contribute  $(n + 1/2)\hbar\omega$  to the total energy, with the ground state at  $n = 0$ . Thus the ground state for the 3-d oscillator will have energy  $3\hbar\omega/2$ , and the general energy level will increase in steps of  $\hbar\omega$  so the energy levels are given by

$$E_n = \left(n + \frac{3}{2}\right)\hbar\omega \quad (4)$$

Unlike the 1-d case, the energies of the 3-d oscillator are degenerate. A given value of  $n$  is composed of the sum of 3 quantum numbers:  $n = n_x + n_y + n_z$  where all numbers are non-negative integers. Suppose we choose a value for  $n_x$  so that  $n_y + n_z = n - n_x$ . The number of pairs of

integers that can be used for  $n_y + n_z$  is  $n - n_x + 1$  (since  $n_y$  can be anything between 0 and  $n - n_x$ ). Since  $n_x$  itself can range between 0 and  $n$ , the total number of combinations of quantum states that can make up state  $n$  is

$$d(n) = \sum_{n_x=0}^n (n - n_x + 1) \quad (5)$$

$$= (n + 1) \sum_{n_x=0}^n 1 - \sum_{n_x=0}^n n_x \quad (6)$$

$$= (n + 1)^2 - \frac{1}{2}n(n + 1) \quad (7)$$

$$= \frac{1}{2}(n + 1)(n + 2) \quad (8)$$

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