

HYDROGEN ATOM: WAVE FUNCTION EXAMPLE 3

Link to: physicspages home page.

To leave a comment or report an error, please use the auxiliary blog.

Reference: Griffiths, David J. (2005), Introduction to Quantum Mechanics, 2nd Edition; Pearson Education - Problem 4.44.

Another example of the hydrogen atom wave function. This time, we'll look at ψ_{433} . Using the formulas in the last post we get $L_0^7(x) = 7! = 5040$, and

$$Y_3^3 = - \left(\frac{35}{64\pi} \right)^{\frac{1}{2}} \sin^3 \theta e^{3i\phi} \quad (1)$$

The wave function is then:

$$\psi_{433} = \sqrt{\left(\frac{1}{2a}\right)^3 \frac{1}{6 \times 5040^3}} e^{-r/4a} \left(\frac{r}{2a}\right)^3 (5040) \left(-\left(\frac{35}{64\pi}\right)^{\frac{1}{2}} \sin^3 \theta e^{3i\phi}\right) \quad (2)$$

$$= -\frac{1}{6144\sqrt{\pi a^9}} r^3 e^{-r/4a} \sin^3 \theta e^{3i\phi} \quad (3)$$

As a check, we verify that this function is normalized:

$$\int_0^{2\pi} \int_0^\pi \int_0^\infty |\psi_{433}|^2 r^2 \sin \theta dr d\theta d\phi = \frac{1}{6144^2 \pi a^9} \int_0^{2\pi} \int_0^\pi \int_0^\infty r^8 e^{-r/2a} \sin^7 \theta dr d\theta d\phi \quad (4)$$

$$= 1 \quad (5)$$

using Maple for the integral.

The expectation value of r is

$$\langle r \rangle = \int_0^{2\pi} \int_0^\pi \int_0^\infty |\psi_{433}|^2 r^3 \sin \theta dr d\theta d\phi \quad (6)$$

$$= \frac{1}{6144^2 \pi a^9} \int_0^{2\pi} \int_0^\pi \int_0^\infty r^9 e^{-r/2a} \sin^7 \theta dr d\theta d\phi \quad (7)$$

$$= 18a \quad (8)$$

As in the previous post, we can generalize this to find the expectation value of r^s :

$$\langle r^s \rangle = \frac{1}{40320} (2a)^s \Gamma(9+s) \quad (9)$$

The gamma function is infinite at all non-positive integral arguments, so this value is finite for $s > -9$ and all non-integer values less than -9 . The smallest integer value for which this is finite is $s = -8$. Note in particular that if $s = 0$, the formula reduces to $\langle r^0 \rangle = \Gamma(9)/40320 = 8!/40320 = 1$ as it should.

Since ψ_{433} is an eigenfunction of L^2 with eigenvalue $l(l+1)\hbar^2 = 12\hbar^2$ and is also an eigenfunction of L_z with eigenvalue $m\hbar = 3\hbar$ the only possible value of $L_x^2 + L_y^2 = L^2 - L_z^2$ is $(12 - 9)\hbar^2 = 3\hbar^2$.